TOPIC 4

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use systems of linear equations and inequalities to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A(n) _____ is made up of two or more inequalities.
3. A(n) _____ is an inequality that is in the same form as a linear equation in two variables, but with an inequality symbol instead of an equal sign.
4. A(n) _____ is an ordered pair that makes all of the inequalities in the system true.
5. The _____ is the set of all ordered pairs that satisfy the inequalities in the system.

- linear inequality in two variables
- solution of an inequality in two variables
- solution of a system of linear inequalities
- system of linear inequalities

Concepts & Skills Review

LESSON 4-1 Solving Systems of Equations by Graphing

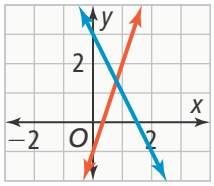
Quick Review

Systems of equations can have one solution, infinitely many solutions, or no solution. Graphing a system of linear equations can result in either an approximate solution or an exact solution.

Example

What is the solution of the system of equations? Use a graph.

$y = 3x - 1$
$y = -2x + 3$



The graph intersects at one point, so the system of linear equations has one solution. Find the point of intersection. The graph intersects at (0.8, 1.4).

Check that the solution works for both equations.

$y = 3x - 1$ $\quad$ $y = -2x + 3$

$1.4 \stackrel{?}{=} 3(0.8) - 1$ $\quad$ $1.4 \stackrel{?}{=} -2(0.8) + 3$

$1.4 = 1.4$ ✓ $\quad$ $1.4 = 1.4$ ✓

The system of equations has one solution at (0.8, 1.4).

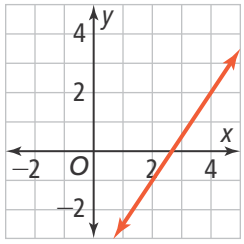
Practice & Problem Solving

Approximate the solution of each system of equations by graphing.

6. $y = 5x + 4$
   $y = -3x - 8$

7. $y = -3x - 7$
   $y = 1.5x + 4$

8. **Use Structure** Describe the solution set of the system of equations made by the equation $y = 1.5x + 4.5$ and the graphed line.



9. **Model With Mathematics** Kiyo is considering two catering companies for a party. A+ Food charges $35 per person and $75 to setup. Super Cater charges $38 per person with no setup fee. Write and solve a system of equations to represent the charges for catering by each company. Which company should Kiyo use if she expects 28 guests?

LESSON 4-2 Solving Systems of Equations by Substitution

Quick Review

To use substitution to solve a system of equations, isolate the variable in one equation and substitute its value into the other equation. Solve for the variable. Then use that value to solve for the other variable.

Example

Solve the system of equations.

$$y = 3x - 5$$
$$4x - 2y = 8$$

Substitute $3x - 5$ for y and solve for x.

$$\begin{aligned} 4x - 2y &= 8 \\ 4x - 2(3x - 5) &= 8 \\ 4x - 6x + 10 &= 8 \\ -2x &= -2 \\ x &= 1 \end{aligned}$$

Substitute 1 for x in either equation and solve for y.

$$\begin{aligned} y &= 3x - 5 \\ &= 3(1) - 5 \\ &= -2 \end{aligned}$$

The solution of the system of equations is $(1, -2)$.

Practice & Problem Solving

Use substitution to solve each system of equations.

10. $y = 5x - 2$
$3x - 5y = 4$

11. $y = 2x - 3$
$y = 8 - 2x$

12. $x = 4y - 8$
$3x - 6y = 12$

13. $y = 2.5x - 8$
$3x + 5y = 12$

Identify whether each system of equations has infinitely many solutions or no solution.

14. $3y = 3x - 9$
$y - 2 = x$

15. $3x - 4y = 12$
$\frac{3}{4}x = y + 3$

16. Mathematical Connections A room has a perimeter of 40 feet. The length is 4 less than 2 times the width. What are the dimensions of the room?

17. Model With Mathematics Benson has 58 more boxed action figures than collector pins. In total he has 246 collectible items. How many of each type of collectible item does Benson own?

LESSON 4-3 Solving Systems of Equations by Elimination

Quick Review

To use elimination to solve a system of equations, multiply one or both equations by a number so that the coefficient of one variable in both equations is the same or opposite. Then add or subtract to eliminate one variable, and solve for the remaining variable.

Example

Solve the system of equations.

$$4x - 3y = 12$$
$$5x - 6y = 18$$

Multiply the first equation by -2 and add the two equations to eliminate y and solve for x.

$$\begin{aligned} 4x - 3y &= 12 \quad \text{Multiply by } -2. \quad -8x + 6y = -24 \\ 5x - 6y &= 18 \qquad\qquad\qquad\quad\ \ 5x - 6y = 18 \\ &\qquad\qquad\qquad\qquad\quad\ \ -3x = -6 \\ &\qquad\qquad\qquad\qquad\quad\ \ x = 2 \end{aligned}$$

Substitute 2 for x into either equation and solve for y.

$$\begin{aligned} 5(2) - 6y &= 18 \\ y &= -\frac{4}{3} \end{aligned}$$

The solution of the system of equations is $\left(2, -\frac{4}{3}\right)$.

Practice & Problem Solving

Solve each system of equations.

18. $2x - y = -2$
$3x - 2y = 4$

19. $5x - 2y = 10$
$4x + 3y = -6$

Is each pair of systems equivalent? Explain.

20. $2x - 3y = 14$
$5x - 2y = 8$

$4x - 6y = 28$
$-15x + 6y = -24$

21. $3x - 4y = -6$
$2x + 5y = 1$

$6x - 8y = 12$
$6x + 15y = 3$

22. Generalize Do you always have to multiply one or both equations to use elimination? Explain.

23. Model With Mathematics Carmen and Alicia go the office supply store to purchase packs of pens and paper. Carmen bought 5 packs of paper and 3 packs of pens for $36.60. Alicia bought 6 packs of paper and 6 packs of pens for $53.40. What is the price of one pack of paper and one pack of pens?

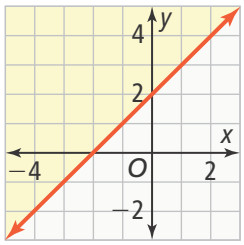
LESSON 4-4 Linear Inequalities in Two Variables

Quick Review

A **linear inequality in two variables** is an inequality that is in the same form as a linear equation in two variables but with an inequality symbol instead of an equal sign. A **solution of a linear inequality in two variables** is an ordered pair that satisfies the inequality.

Example

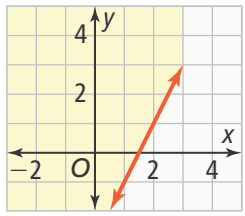
What inequality is shown by the graph?



The slope of the line is 1 and its y-intercept is 2. Therefore, the equation of the line is $y = x + 2$. The boundary line is solid, and all values of x and y that make the inequality true lie on the line or above the line. The inequality shown by the graph is $y \geq x + 2$.

Practice & Problem Solving

Use the graph to tell whether each ordered pair is a solution of the inequality $y \geq 2x - 3$.



24. (2, 5)

25. (3, −1)

26. (−2, 4)

Graph the inequality in the coordinate plane.

27. $y > 4x - 9$

28. $y \leq 1.5x + 4$

29. **Reason** Write an inequality in two variables for which (2, 5) and (−3, −1) are solutions.

30. **Make Sense and Persevere** Renaldo has a budget of $500 to buy gift boxes for a party. Large boxes cost $65 and small boxes cost $35. Write and graph an inequality that represents the number of each type of gift box that Renaldo can buy. If Renaldo buys 6 small gift boxes, how many large gift boxes can he afford to buy?

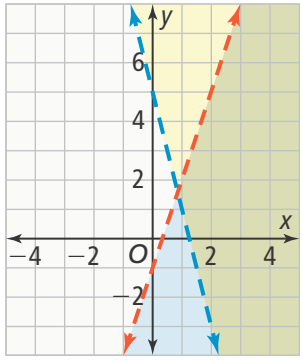
LESSON 4-5 Systems of Linear Inequalities

Quick Review

A **system of linear inequalities** is made up of two or more inequalities. The **solutions of a system of linear inequalities** is the set of all ordered pairs that satisfy the inequalities in the system.

Example

What system of inequalities is shown by the graph? Describe the solutions of the system of inequalities.



Determine the equation of each line using the slope and y-intercept.

Points below the red dashed line satisfy the inequality $y < 3x - 1$. Points above the blue dashed line satisfy the inequality $y > -4x + 5$.

The solutions to the system lie in the region where the graphs overlap.

Practice & Problem Solving

Graph each system of inequalities.

31. $y < 2x + 3$
$y \leq -3x + 1$

32. $y \geq 4x$
$y < -x - 5$

33. **Generalize** What two inequalities can you add to any system of inequalities to indicate that only answers in the first quadrant make sense?

34. **Model With Mathematics** Olivia makes and sells bracelets and necklaces. She can make up to 60 pieces per week, but she can only make up to 40 bracelets and 40 necklaces. Write and graph a system of inequalities that shows the combination of bracelets and necklaces that she can make if she wants to sell at least 30 items per week. If necklaces sell for $80 each and bracelets sell for $5 each, what is the most money she can make in a week? Explain.