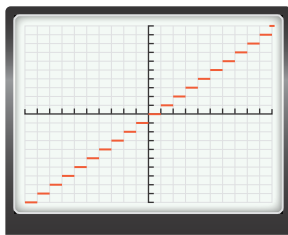**UNDERSTAND**

**10. Communicate Precisely** Many calculators use an INT function which returns the greatest integer less than or equal to $x$. The graph of **Y1 = INT(X)** is shown. How is this function like the floor function? How is it different?

**11. Look for Relationships** How are the pieces of a step function related to the domain of the function? Justify your thinking.

**12. Error Analysis** Kenji wrote a step function to round numbers up to nearest multiple of three. Describe and correct the error he made.

$$f(x) = \begin{cases} 3, & 3 < x \le 6 \\ 6, & 6 < x \le 9 \\ 9, & 9 < x \le 12 \\ 12, & 12 < x \le 15 \end{cases} \quad ✗$$

**13. Communicate Precisely** Explain how you can use the graph shown below to find the value of the step function for $x = 1$. How is this different from finding the value for $x = 1$ when the graph of a function is a straight line?
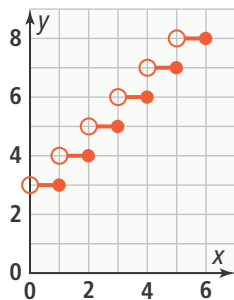
**14. Higher Order Thinking** Results of the **INT** function are shown in the spreadsheet.

a. If $f(x) = \text{INT}(x)$, what is $f(4.6)$, $f(5)$, and $f(-6.5)$?

b. Write $f(x) = \text{INT}(x)$ as a step function for the domain $-4 \le x \le 4$.

|   | A | B |
|---|---|---|
| 1 | −3.1 | =INT(A1) |
| 2 | −2.4 | −3 |
| 3 | −1.8 | −2 |
| 4 | −0.9 | −1 |
| 5 | 0 | 0 |
| 6 | 0.8 | 0 |
| 7 | 1.9 | 1 |
| 8 | 2.8 | 2 |

**PRACTICE**

**Evaluate the function for the given value.**

SEE EXAMPLE 1

**15.** $f(x) = \lceil x \rceil$; $x = 0.1$

**16.** $f(x) = \text{ceiling}(x)$; $x = 5.15$

**17.** $f(x) = \lfloor x \rfloor$; $x = -4.01$

**18.** $f(x) = \text{ceiling}(x)$; $x = 13.20$

**19.** $f(x) = \lfloor x \rfloor$; $x = 7.06$

**20.** $f(x) = \text{floor }(x)$; $x = 33.7$

**21.** $f(x) = \text{floor }(x)$; $x = 23.2$

**22.** $f(x) = \lfloor x \rfloor$; $x = -8.4$

**For each table, graph the step function and write a rule for $f$ using the ceiling or floor function.**

SEE EXAMPLES 2 AND 3

**23.**

| x | f(x) |
|---|------|
| $0 < x \le 1$ | 5 |
| $1 < x \le 2$ | 6 |
| $2 < x \le 3$ | 7 |
| $3 < x \le 4$ | 8 |
| $4 < x \le 5$ | 9 |
| $5 < x \le 6$ | 10 |

**24.**

| x | f(x) |
|---|------|
| $0 \le x < 2$ | 3 |
| $2 \le x < 4$ | 4 |
| $4 \le x < 6$ | 5 |
| $6 \le x < 8$ | 6 |
| $8 \le x < 4$ | 7 |
| $4 \le x < 12$ | 8 |

**Sketch the graph of each function over the domain $0 < x \le 10$.**

**25.** The function $g$ returns the greatest integer $g(x)$ that is less than or equal to $x + 2$.

**26.** The function $f$ returns the least integer $f(x)$ that is greater than $3x$.

# PRACTICE & PROBLEM SOLVING

**APPLY**

**27. Mathematical Connections** There are 240 seniors in Kathryn's school. Her class is planning a trip, and is taking buses that hold a maximum of 50 passengers. Assume that the trip is optional.

a. Write a step function $f$ that maps the number of students $x$, to the number of buses needed, $f(x)$.

b. What assumptions to do you need to make to write the function?

c. What is the average rate of change of the function over the interval from 40 to 60? From 60 to 80?

d. What do the average rates of change mean in terms of the situation? Explain.

**28. Construct Arguments** Amit parked his car for 144 h, and Nan parked her car for 145 h. Will Nan pay more? If so, how much? Make a table and then graph a function to support your answer.
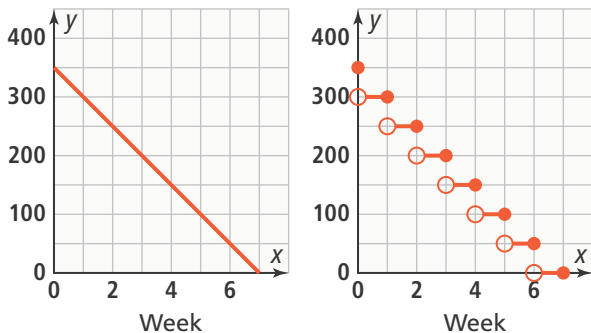
**Airport Parking Rates** ✈

$50 for first 24 hours plus $25 for each additional 24 hours.

Any fraction of a 24-hour period will be charged for the entire 24-hour period.

**29. Model With Mathematics** Mia has $350 in her bank account at the beginning of the school year. Every week she withdraws $50. Two graphs model the situation.



a. Write a function for each graph.

b. How do the graphs and the functions differ in how they represent the situation?

c. What are the advantages and disadvantages of each type of function?

**ASSESSMENT PRACTICE**

**30.** A resort rents skis for $15 for the first hour and $7.50 for each additional hour. Copy and complete the table for the step function that models the total cost, in dollars, of renting skis for $x$ hours.

| $x$ | $f(x)$ |
|---|---|
| _____ | 15 |
| $1 < x \le 2$ | _____ |
| _____ | _____ |
| $3 < x \le 4$ | _____ |
| _____ | 45 |
| $5 < x \le 6$ | _____ |

**31. SAT/ACT** What is the value of $f(2) + f(4) + f(11) + f(12)$ for the function $f$?

$$f(x) = \begin{cases} 100, & 0 < x \le 4 \\ 95, & 4 < x \le 8 \\ 90, & 8 < x \le 12 \\ 95, & 12 < x \le 16 \end{cases}$$

Ⓐ 30          Ⓑ 280

Ⓒ 290          Ⓓ 380

Ⓔ 300

**32. Performance Task** Abdul and his family are traveling on a toll highway. The table shows the cost of using the highway as a function of distance.

| Exit Number | Distance (mi) | Toll ($) |
|---|---|---|
| 1 | 0 | 0.00 |
| 2 | 40 | 1.25 |
| 3 | 75 | 1.75 |
| 4 | 85 | 1.90 |
| 5 | 120 | 2.25 |
| 6 | 150 | 2.50 |

**Part A** Write a step function $t$ to represent the cost of the tolls in terms of distance.

**Part B** Assume their car averages 30 mi/gal and gasoline costs $3.50/gal. Write a function $g$ to represent the cost of the gas in terms of distance.

**Part C** Use functions $t$ and $g$ to determine the cost of Abdul's trip if his family leaves the highway at Exit 5.