TOPIC 5

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use piecewise-defined functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The _____ intersects the vertex, and divides the graph into two congruent halves that are images of each other under a reflection.
3. The _____ rounds numbers up to the nearest integer.
4. The _____ has an algebraic expression with absolute value symbols.
5. A(n) _____ has different rules for different intervals of its domain.

- absolute value function
- axis of symmetry
- ceiling function
- floor function
- piecewise-defined function
- step function
- vertex

Concepts & Skills Review

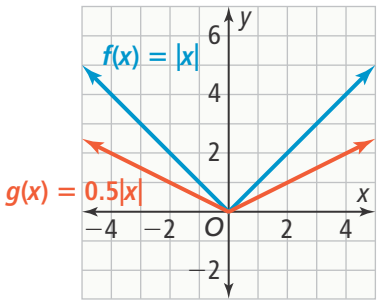
LESSON 5-1 The Absolute Value Function

Quick Review

The graph of the absolute value function, $f(x) = |x|$ has a **vertex** at (0, 0) and an **axis of symmetry** $x = 0$.

Example

How do the domain and range of $g(x) = 0.5|x|$ compare with the domain and range of $f(x) = |x|$?



The domain of f and the domain of g are all real numbers. The range of both functions is $y \geq 0$.

Practice & Problem Solving

Tell whether each point is on the graph of $f(x) = |x|$. If it is, give the coordinates of the another point with the same y-coordinate.

6. (8, 8)
7. (−5, 5)
8. (−3.5, −3.5)

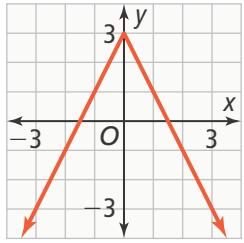
Graph each function. What is the domain and range of each function?

9. $f(x) = -2.5|x|$
10. $g(x) = \frac{1}{3}|x|$

For each function, find the vertex and tell whether it represents a maximum or minimum value of the function.

11. $g(x) = -6.3|x|$
12. $g(x) = 7|x|$

13. **Look for Relationships** Find the domain, range, and vertex of the graphed function.



LESSON 5-2 Piecewise-Defined Functions

Quick Review

A **piecewise-defined function** has different rules for different intervals of the domain.

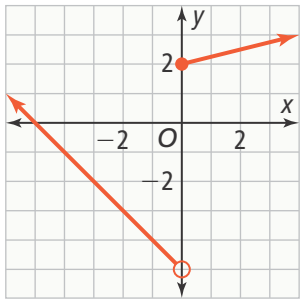
You can express functions of the form $g(x) = a|x|$ as piecewise-defined functions using a pair of linear rules with boundaries on the domain.

Example

What is the graph of $f(x) = \begin{cases} -x - 5, & x < 0 \\ \frac{1}{4}x + 2, & x \geq 0 \end{cases}$?

Over what interval of the domain is the function increasing? Decreasing?

Graph each rule of the function for the given interval of the domain.



The function is decreasing when $x < 0$.
The function is increasing when $x \geq 0$.

Practice & Problem Solving

Express each function as a piecewise-defined function.

14. $f(x) = 4|x|$

15. $f(x) = -2|x|$

Graph each piecewise-defined function. Identify the intervals over which the function is increasing, decreasing, or constant.

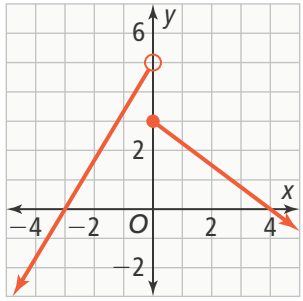
16. $f(x) = \begin{cases} x + 1, & x < -1 \\ \frac{1}{2}x - 2, & -1 \leq x \end{cases}$

17. $f(x) = \begin{cases} -x + 4, & -2 < x \leq 3 \\ x - 5, & 3 < x \end{cases}$

18. Error Analysis Describe and correct the error a student made in expressing the function $f(x) = -5|x|$ as a piecewise-defined function.

$$f(x) = -5|x|,\ f(x) = \begin{cases} -5x, & x < 0 \\ 5x, & x > 0 \end{cases}$$

19. Write a piecewise-defined function that represents the graph.



20. Model With Mathematics A jeweler sells rings for $35 each plus a flat fee of $5 for shipping for orders up to 10 rings. If customers order more than 10 rings, the cost is $30 per ring and shipping is free. Write a piecewise function to represent the situation.

Quick Review

Step functions are a type of piecewise-defined function that consists of constant pieces. The constant pieces of the function result in a graph that looks like the steps of a staircase.

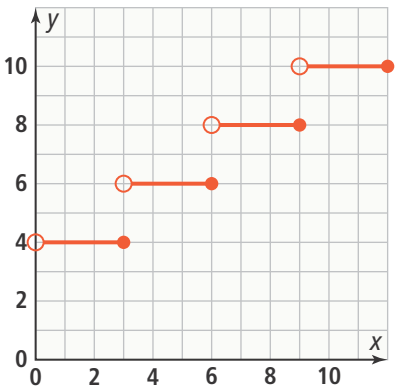
The floor and ceiling functions are specific types of step functions. The **ceiling function** rounds numbers up to the nearest integer. The **floor function** rounds numbers down to the nearest integer.

Example

Graph the function *f*. What is the domain and range of the function?

x	$f(x)$
$0 < x \leq 3$	4
$3 < x \leq 6$	6
$6 < x \leq 9$	8
$9 < x \leq 12$	10

Each section of the domain has a single value assigned to it.



The domain of f is $0 < x \leq 12$. The range is the set values {4, 6, 8, 10}.

Practice & Problem Solving

Evaluate the ceiling function for the given value.

21. $f(x) = \lceil x \rceil$; $x = 7.03$

22. $f(x) = \text{ceiling}(x)$; $x = 2.6$

Evaluate the floor function for the given value.

23. $f(x) = \lfloor x \rfloor$; $x = 6.1$

24. $f(x) = \text{floor}(x)$; $x = 0.08$

For each table, graph the step function and write a rule for *f* using a ceiling or floor function.

25.

x	$f(x)$
$1 < x \leq 2$	7
$2 < x \leq 3$	8
$3 < x \leq 4$	9
$4 < x \leq 5$	10
$5 < x \leq 6$	11

26.

x	$f(x)$
$0 < x \leq 2$	-2
$2 < x \leq 4$	-3
$4 < x \leq 6$	-4
$6 < x \leq 8$	-5
$8 < x \leq 10$	-6

Sketch the graph of each function over the domain $0 < x \leq 10$.

27. The function g returns the greatest integer $g(x)$ less than or equal to $2x$.

28. The function f returns the least integer $f(x)$ greater than $x - 3$.

29. **Make Sense and Persevere** Egg cartons hold a dozen eggs in each container. Write a step function that represents the number of egg cartons needed as a function of the number of eggs over the domain $0 < x \leq 72$. Is the function a floor or a ceiling function? Explain.

Quick Review

The graph of $g(x) = a|x - h| + k$ is a transformation of the graph of $f(x) = |x|$ when $a \neq 1$, $h \neq 0$, or $k \neq 0$. The vertex of the graph is located at (h, k). The value of h indicates that the graph of g is a horizontal translation of h units of the graph of f. The value of k indicates that the graph of g is a vertical translation of k units of the graph of f.

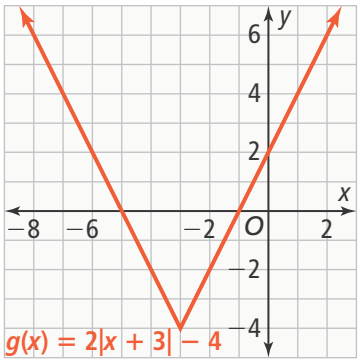
When $|a| > 1$, the graph of g is a vertical stretch of the graph of f. When $0 \leq |a| \leq 1$, the graph is a vertical compression of the graph of f.

Example

For the function $g(x) = 2|x + 3| - 4$, find the vertex and graph the function. Describe the graph of g as a transformation of the graph of $f(x) = |x|$.

The vertex is located at (h, k), so the vertex of this graph is $(-3, -4)$.

Graph the function.



The graph of g is a translation of the graph of f horizontally, 3 units to the left, and vertically 4 units down. It is also a vertical stretch of the graph of f by a factor of 2.

Practice & Problem Solving

Find the vertex and graph each function.

30. $g(x) = |x| + 4$

31. $g(x) = |x - 2|$

32. $g(x) = |x + 1| - 2$

33. $g(x) = |x - 3| + 1$

Compare each function with $f(x) = |x|$. Describe the graph of g as transformation of the graph of f.

34. $g(x) = 2|x + 6| - 1$

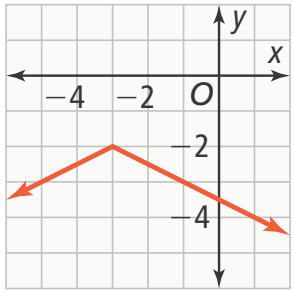
35. $g(x) = -|x - 2| - 1$

36. $g(x) = -0.5|x| + 4$

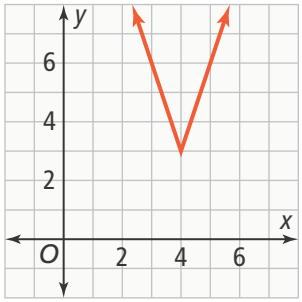
37. $g(x) = \frac{3}{2}|x - 1| + 8$

Write the function that includes absolute value expressions for each graph.

38.



39.



40. **Use Structure** Write two functions that have a vertex of $(2, -1)$.

What function g describes the graph of f after the given transformations?

41. $f(x) = |x| + 3$; translated 1 unit up and 5 units right

42. $f(x) = |x - 1| + 5$; translated 2 units down and 4 units left

43. **Make Sense and Persevere** A traffic cone is 18 in. tall and 12 in. wide. You want to sketch an image of the traffic cone on a coordinate grid with one edge at (0, 0). What function that includes an absolute value expression could represent the traffic cone? What would be the domain of the function? Explain.