TOPIC 6

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use exponential functions to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. A population's growth can be modeled by a(n) __________ function of the form $f(x) = a \cdot b^x$, where $a > 0$ and $b > 1$.
3. An exponential function repeatedly multiplies an initial amount by the same positive number, called the __________.
4. A(n) __________ is a number sequence formed by multiplying a term in the sequence by a fixed nonzero number, or a common ratio, to find the next term.
5. __________ is interest that is paid both on the principal and on the interest that has already been paid.
6. As x or y gets larger in absolute value, the graph of the exponential function gets closer to the line called a(n) __________.

- geometric sequence
- constant ratio
- simple interest
- decay factor
- compound interest
- exponential decay
- exponential growth
- exponential function
- asymptote
- growth factor

Concepts & Skills Review

LESSON 6-1 Rational Exponents and Properties of Exponents

Quick Review

If the nth root of a is a real number and m is an integer, then $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$.

Power of a Power: $(a^m)^n = a^{mn}$

Power of a Product: $(a \cdot b)^m = a^m b^m$

Product of Powers: $a^m \cdot a^n = a^{m+n}$

Quotient of Powers: $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$

Example

How can you use the Power of a Power Property to solve $64^{x-3} = 16^{2x-1}$?

Rewrite the equation so both expressions have the same base.

$$64^{x-3} = 16^{2x-1}$$
$$(2^6)^{x-3} = (2^4)^{2x-1}$$
$$2^{6x-18} = 2^{8x-4}$$
$$6x - 18 = 8x - 4$$
$$-18 = 2x - 4$$
$$-14 = 2x$$
$$-7 = x$$

The solution is –7.

Practice & Problem Solving

Write each radical using rational exponents.

7. $\sqrt{8}$
8. $\sqrt[3]{12}$

Solve each equation.

9. $\left(6^{\frac{x}{2}}\right)\left(6^{\frac{x}{3}}\right) = 6^6$
10. $36^{4x-1} = 6^{x+2}$
11. **Make Sense and Persevere** Describe two ways to express the edge length of a cube with a volume of 64 cm^3.
12. **Model With Mathematics** Use rational exponents to express the relationship between the dollar values of two prizes in a contest.

Prize	Value
Bicycle	$256
Luxury vehicle	$65,536

TOPIC 6 REVIEW

LESSON 6-2 Exponential Functions

Quick Review

An exponential function is the product of an initial amount and a **constant ratio** raised to a power. Exponential functions are expressed using $f(x) = a \cdot b^x$, where a is a nonzero constant, $b > 0$, and $b \neq 1$.

Example

Find the initial amount and the constant ratio of the exponential function represented by the table.

x	$f(x)$	
0	3	The initial amount is 3.
1	12	$12 \div 3 = 4$
2	48	$48 \div 12 = 4$
3	192	$192 \div 48 = 4$
4	768	$768 \div 192 = 4$

The constant ratio is 4.

In $f(x) = a \cdot b^x$, substitute 3 for a and 4 for b.

The function is $f(x) = 3(4)^x$.

Practice & Problem Solving

Graph each exponential function.

13. $f(x) = 2.5^x$

14. $f(x) = 5(2)^x$

15. Write the exponential function for this table.

x	0	1	2	3
$f(x)$	0.5	1	2	4

16. **Make Sense and Persevere** Write an equation for an exponential function that models the expected number of bacteria as a function of time. Graph the function. If the pattern continues, in which month will the bacteria exceed 45,000,000?

Month	Number of Bacteria
0	2,500
1	7,500
2	22,500
3	67,500
4	202,500

LESSON 6-3 Exponential Growth and Decay

Quick Review

An **exponential growth function** can be written as $f(x) = a(1 + r)^x$. An exponential decay function can be written as $f(x) = a(1 - r)^x$.

Example

Chapter City has a population of 18,000 and grows at an annual rate of 8%. What is the estimated population of Chapter City in 6 years?

Let x = time in years, a = initial amount, and r = growth rate.

$$f(x) = a(1 + r)^x$$
$$= 18{,}000(1 + 0.08)^x$$

The function is $f(x) = 18{,}000(1.08)^x$.

Find the expected population in 6 years.

$f(6) = 18{,}000(1.08)^6 \approx 28{,}563.74$

After 6 years, the population is expected to be about 28,564.

Practice & Problem Solving

17. **Make Sense and Persevere** An exponential function of the form $f(x) = b^x$ includes the points (2, 36), (3, 216), and (4, 1,296). What is the value of b?

Write an exponential growth or decay function to model each situation.

18. initial value: 50, growth factor: 1.15

19. initial value: 200, decay factor: 0.85

Construct Arguments **Compare each investment to an investment of the same principal at the same rate compounded annually.**

20. principal: \$12,000
annual interest: 5%
interest periods: 2
number of years: 10

21. principal: \$20,000
annual interest: 2.5%
interest periods: 4
number of years: 15

LESSON 6-4 Geometric Sequences

Quick Review

A **geometric sequence** is a number sequence in which each term after the first term is found by multiplying the previous term by a common ratio.

Explicit formula: $a_n = a_1(r)^{n-1}$

Recursive formula: $a_n = r(a_{n-1})$

Example

What are the explicit and recursive formulas for the geometric sequence 9, 22.5, 56.25, 140.625, 351.5625, … ?

$\frac{22.5}{9} = \frac{56.25}{22.5} = \frac{140.625}{56.25}$

$= \frac{351.5625}{140.625} = \frac{5}{2}$ ······ Find the common ratio.

The common ratio is $\frac{5}{2}$. The first term is 9.

The explicit formula is $a_n = 9\left(\frac{5}{2}\right)^{n-1}$

The recursive formula is $a_n = \frac{5}{2}(a_{n-1})$, $a_1 = 9$.

Practice & Problem Solving

Determine if the sequence is a geometric sequence. If it is, write the explicit and recursive formulas.

22. $5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \ldots$

23. 2, 5, 8, 11, 14, …

24. 8, 16, 32, 64, 128, …

25. $\frac{1}{5}, \frac{2}{5}, \frac{4}{5}, \frac{8}{5}, \frac{16}{5}, \ldots$

Translate each explicit formula to recursive form.

26. $a_n = 2.2(4)^{n-1}$

27. $a_n = 6(3.5)^{n-1}$

28. Write the explicit and recursive formula for a geometric sequence modeled in the table. Will the number of signatures reach 7,000 by the end of the second week? Explain.

Petition to Turn Parking Lot into Park

Day	Number of Signatures
1	40
2	60
3	90
4	135
5	202.5

LESSON 6-5 Transformations of Exponential Functions

Quick Review

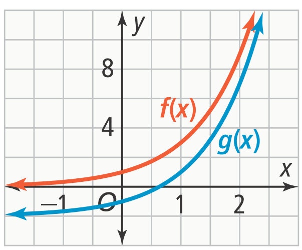
The graph of $g(x) = a^x + k$ is the graph of a^x translated up when $k > 0$ and translated down when $k < 0$.

The graph of $g(x) = a^{x-h}$ is the graph of a^x translated right when $h > 0$ and translated left when $h < 0$.

Example

Compare the graphs of $g(x) = 3^x - 2$ and $f(x) = 3^x$.

x	$f(x)$	$g(x)$
−2	$\frac{1}{9}$	$-\frac{17}{9}$
−1	$\frac{1}{3}$	$-\frac{5}{3}$
0	1	−1
1	3	1
2	9	7



The graph of $g(x)$ is translated 2 units down from the graph of $f(x)$.

Practice & Problem Solving

Compare the graph of each function to the graph of $f(x) = 3^x$.

29. $g(x) = 3^x - 5$

30. $j(x) = 3^x + 10$

31. $g(x) = 3^{x-2}$

32. $j(x) = 3^{x+3}$

Graph each function and its transformation.

33. $f(x) = 1.5^x$, $g(x) = 1.5^x + k$ for $k = 2$

34. $f(x) = 4^x$, $g(x) = 4^x - k$ for $k = 0.5$