LESSON 7-4 Factoring Polynomials

Quick Review

To factor a common monomial factor out of a polynomial, first write the prime factorization of the coefficient for each term to determine if there is a greatest common factor other than 1. Then determine the greatest common factor for the variables of each term.

Example

What is the GCF of the terms of $16x^6 - 8x^4 + 4x^3$?

First, write the prime factorization of the coefficients for each term.

$16 = 2 \cdot 2 \cdot 2 \cdot 2$
$8 = 2 \cdot 2 \cdot 2$
$4 = 2 \cdot 2$

Each number has a common coefficient of 4, so the GCF of the coefficients is 4.

Next, determine the GCF of the variables for each term.

$x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$
$x^4 = x \cdot x \cdot x \cdot x$
$x^3 = x \cdot x \cdot x$

Each term has the common factor of x^3, so the GCF of the variables is x^3.

The GCF of $16x^6 - 8x^4 + 4x^3$ is $4x^3$.

Practice & Problem Solving

Find the GCF of each group of monomials.

29. $6x^2$, $21x$
30. bc^2, b^3c
31. $14x^2y^2$, $84x^3y^5$, $21xy^3$
32. $24a^2$, 18

Factor out the GCF from each polynomial.

33. $15x^3 - 42x$
34. $16y^5 - 42y^3 + 18y$
35. $12a^3 + 18a^2 - 36a$
36. $49a^5b^3 - 14a^2b^2 + 35ab$

37. **Look for Relationships** Write a trinomial that has a GCF of $3x$.

38. **Use Structure** Determine the GCF and write the expression in factored form.

$(8x^2 - 12x) + (6x^2 - 4x)$

LESSON 7-5 Factoring $x^2 + bx + c$

Quick Review

To factor $x^2 + bx + c$, find the factor pair of c that has a sum of b. Then use those factors to write the binomial factors of the trinomial.

Example

What is the factored form of $x^2 - 9x + 14$?

Identify a factor pair of 14 that has a sum of -9.

Factors of 14	Sum of Factors
-1 and -14	-15
-2 and -7	-9

The factored form of $x^2 - 9x + 14$ is $(x - 2)(x - 7)$.

Practice & Problem Solving

Complete the table to factor the trinomial

39. $x^2 + 7x - 18$

Factors of c	Sum of Factors
-1 and 18	17
1 and -18	-17
9 and -2	7

Write the factored form of each trinomial.

40. $x^2 + 12x + 32$
41. $x^2 + 3x - 28$
42. $x^2 - 13x - 48$
43. $x^2 + 18xy + 45y^2$

44. **Look for Relationships** How are the binomial factors of $x^2 + 4x - 21$ and $x^2 - 4x - 21$ similar? How are they different?

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LESSON 7-6 Factoring $ax^2 + bx + c$

Quick Review

To factor a trinomial of the form $ax^2 + bx + c$, find the factor pair of ac that has a sum of b. Then use the factors you found to write the binomials that have a product equal to the trinomial.

Example

What is the factored form of $2x^2 + 9x - 5$?

For the trinomial $2x^2 + 9x - 5$, $a = 2$ and $c = -5$, so $ac = -10$. Find the factor pair of -10 that has a sum of 9.

Factors of -10	Sum of Factors
-2 and 5	3
2 and -5	-3
-1 and 10	9

Since -1 and 10 are the correct factor pair, rewrite $9x$ as $-1x$ and $10x$.

$2x^2 + 9x - 5$
$= 2x^2 + 10x - 1x - 5$ ····· Rewrite.
$= (2x^2 + 10x) + (-1x - 5)$ ····· Group as two binomials.
$= 2x(x + 5) - 1(x + 5)$ ····· Factor out the GCFs.

Practice & Problem Solving

Identify all of the factor pairs of ac you could use to rewrite b in order to factor each trinomial by grouping.

45. $5x^2 + 9x + 4$
46. $2x^2 + x - 15$

Write the factored form of each trinomial.

47. $3x^2 + 10x + 8$
48. $4x^2 - 3x - 10$
49. $5x^2 + 7x - 6$
50. $6x^2 + 13x + 6$
51. $10x^2 + 3x - 4$
52. $12x^2 + 22x + 6$

53. **Make Sense and Persevere** What are all the possible values of b for which $3x^2 + bx - 8$ is factorable using only integer coefficients and constants?

54. **Reason** A parking lot has an area of $6x^2 + 27x - 15$ square meters. Use factoring to find possible dimensions of the parking

$2x^2 + 9x - 5$

$= 2x^2 + 10x - 1x - 5$ ..... Rewrite.

$= (2x^2 + 10x) + (-1x - 5)$ ..... Group as two binomials.

$= 2x(x + 5) - 1(x + 5)$ ..... Factor out the GCFs.

$= (2x - 1)(x + 5)$ ..... Distributive Property

The factored form of $2x^2 + 9x - 5$ is $(2x - 1)(x + 5)$.

constants?

54. Reason A parking lot has an area of $6x^2 + 27x - 15$ square meters. Use factoring to find possible dimensions of the parking lot. The parking lot is to be enlarged so that each dimension is 5 meters greater than it was originally. What are the new dimensions of the parking lot? What is the new area of the parking lot?

$2(2x+3)$

LESSON 7-7 Factoring Special Cases

Quick Review

A **perfect-square trinomial** results when a binomial is squared.

Factor a perfect-square trinomial:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Use these patterns when the first and last terms are perfect squares and the middle term is twice the product of the numbers being squared.

Factor a difference of two squares:

$a^2 - b^2 = (a + b)(a - b)$

Use this pattern when a binomial can be written as a difference of two squares.

Example

What is the factored form of $9x^2 - 121$?

Write the first and last term as a perfect square.

$9x^2 - 121 = (3x)^2 - 11^2$

$= (3x - 11)(3x + 11)$

Practice & Problem Solving

Identify the value of c that would make each trinomial factorable using the perfect-square pattern.

55. $x^2 + 16x + c$

56. $2x^2 - 28x + c$

Write the factored form of each expression.

57. $x^2 + 10x + 25$

58. $x^2 - 121$

59. $x^2 - 18x + 81$

60. $9x^2 - 49y^2$

61. $3x^2 + 18x + 27$

62. $4x^2 - 56x + 196$

63. Reason Is the expression $3x^2 - 49$ factorable using only integer coefficients and constants? Explain why or why not.

64. Make Sense and Persevere The area of a playground is $36x^2 - 16y^2$ square feet. Use factoring to find possible dimensions of the playground. How are the side lengths related? What value would you need to subtract from the longer side and add to the shorter side for the playground to be a square?

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