TOPIC 7

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you work with polynomials to rewrite expressions and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The _____ states that polynomials are closed under addition or subtraction because the result of these operations is another polynomial.
3. A(n) _____ results when a binomial is squared.
4. A(n) _____ is a real number, a variable, or the product of a real number and one or more variables with whole number exponents.
5. The product of two binomials in the form $(a + b)(a - b)$ is $a^2 - b^2$, which is called the _____.
6. The _____ is a an expression in which the terms are written in descending order according to their degree.

- Closure Property
- degree of a monomial
- degree of a polynomial
- difference of two squares
- monomial
- perfect-square trinomial
- polynomial
- standard form of a polynomial

Concepts & Skills Review

LESSON 7-1 Adding and Subtracting Polynomials

Quick Review

A **polynomial** is a monomial or the sum or difference of two or more monomials, called terms. Polynomials are named according to their degree. The **degree of a polynomial** is the greatest degree of any term of the polynomial. The **standard form of a polynomial** is a polynomial in which terms are written in descending order according to their degree.

Example

What is the difference $(5x^2 + 3x - 5) - (2x^2 + 8)$?

$(5x^2 + 3x - 5) - (2x^2 + 8)$

$= 5x^2 + 3x - 5 - 2x^2 - 8$ ..... Apply subtraction to each term in the second expression.

$= (5x^2 - 2x^2) + (3x) + (-5 - 8)$ ..... Use the Commutative and Associative Properties to group like terms.

$= 3x^2 + 3x - 13$ ..... Simplify.

The difference is $3x^2 + 3x - 13$.

Practice & Problem Solving

Name each monomial based on its degree.

7. $2xy$
8. -6
9. $3x^2y$

Add or subtract to simplify each expression. Write your final answer in standard form.

10. $(5x - 1) + (2x - 3)$
11. $(2x^2 - 4x - 1) - (3x^2 + 8x - 4)$
12. $(5b^4 - 2 + 3b^2) + (5b^2 - 4 + 3b^4)$
13. **Reason** What is the missing term in the equation? $(\_\_\_ + 5) + (3x - 2) = 8x + 3$. Explain.
14. **Make Sense and Persevere** A garden center has $(3x^2 + 12x + 18)$ sq. ft of sod. One week, they receive $(4x^2 + 16x + 60)$ sq. ft of sod, and sell $(2x^2 + 9x + 27)$ sq. ft of sod. What expression represents the area of the remaining sod?

LESSON 7-2 Multiplying Polynomials

Quick Review

Use the Distributive Property to multiply polynomials as you would when multiplying integers numbers. Distribute the first polynomial to each term in the second polynomial.

Example

How can you use the Distributive Property to rewrite $(3x - 5)(4x - 9)$ as a polynomial?

Distribute the first binomial to each term in the second binomial.

$(3x - 5)(4x - 9)$

$= 3x(4x - 9) - 5(4x - 9)$ ····· Distribute $3x$ and -5 to the second binomial.

$= 3x(4x) + 3x(-9) - 5(4x) - 5(-9)$ ····· Distribute $3x$ and -5 to each term in the second binomial.

$= 12x^2 - 27x - 20x + 45$ ····· Multiply.

$= 12x^2 - 47x + 45$ ····· Combine like terms.

The product is $12x^2 - 47x + 45$.

Practice & Problem Solving

Use the Distributive Property to find each product.

15. $(x + 7)(x - 5)$ **16.** $(2x - 5)(3x + 1)$

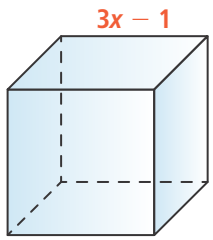
Use a table to find each product.

17. $(4x - 3y)(5x + y)$ **18.** $(x + 4)(x^2 - 3x - 1)$

19. Make Sense and Persevere Identify the missing terms in the quotient and divisor.

$(\_\_\_\_ + 3)(x + \_\_\_\_) = x^2 + 11x + 24$

20. Model With Mathematics The volume of a cube is calculated by multiplying the length, width and height. What is the volume of this cube in standard form?



LESSON 7-3 Multiplying Special Cases

Quick Review

The square of a binomial always follows the same pattern, $a^2 + 2ab + b^2$. The product of two binomials in the form $(a + b)(a - b)$ is $a^2 - b^2$. This is called the **difference of two squares**.

Example

What is the product $(4x - 9)(4x + 9)$?

Use the pattern.

$(4x - 9)(4x + 9)$

$= (4x)^2 - (9)^2$ ····· Substitute $4x$ and 9 and for a and b in $a^2 - b^2$.

$= 16x^2 - 81$ ····· Simplify.

The product is $16x^2 - 81$.

Practice & Problem Solving

Write each product in standard form.

21. $(b + 12)(b + 12)$ **22.** $(4x + 1)(4x + 1)$

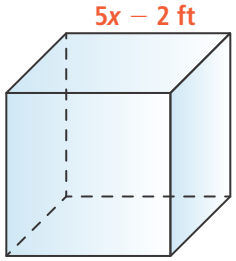
23. $(6x - 9)(6x + 9)$ **24.** $(3x - 4y)(3x + 4y)$

25. $(1.5x + 2)(1.5x - 2)$ **26.** $(3a - 5b)^2$

27. Look for Relationships Find a value for m to make a true statement.

$mx^2 - 64 = (5x + 8)(5x - 8)$

28. Modeling With Mathematics Write polynomials in standard form to represent the surface area and volume of the cube.



LESSON 7-4 Factoring Polynomials

Quick Review

To factor a common monomial factor out of a polynomial, first write the prime factorization of the coefficient for each term to determine if there is a greatest common factor other than 1. Then determine the greatest common factor for the variables of each term.

Example

What is the GCF of the terms of $16x^6 - 8x^4 + 4x^3$?

First, write the prime factorization of the coefficients for each term.

$16 = 2 \cdot 2 \cdot 2 \cdot 2$
$8 = 2 \cdot 2 \cdot 2$
$4 = 2 \cdot 2$

Each number has a common coefficient of 4, so the GCF of the coefficients is 4.

Next, determine the GCF of the variables for each term.

$x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$
$x^4 = x \cdot x \cdot x \cdot x$
$x^3 = x \cdot x \cdot x$

Each term has the common factor of x^3, so the GCF of the variables is x^3.

The GCF of $16x^6 - 8x^4 + 4x^3$ is $4x^3$.

Practice & Problem Solving

Find the GCF of each group of monomials.

29. $6x^2, 21x$

30. bc^2, b^3c

31. $14x^2y^2, 84x^3y^5, 21xy^3$

32. $24a^2, 18$

Factor out the GCF from each polynomial.

33. $15x^3 - 42x$

34. $6y^5 - 42y^3 + 18y$

35. $12a^3 + 18a^2 - 36a$

36. $49a^5b^3 - 14a^2b^2 + 35ab$

37. **Look for Relationships** Write a trinomial that has a GCF of $3x$.

38. **Use Structure** Determine the GCF and write the expression in factored form.

$(8x^2 - 12x) + (6x^2 - 4x)$

LESSON 7-5 Factoring $x^2 + bx + c$

Quick Review

To factor $x^2 + bx + c$, find the factor pair of c that has a sum of b. Then use those factors to write the binomial factors of the trinomial.

Example

What is the factored form of $x^2 - 9x + 14$?

Identify a factor pair of 14 that has a sum of –9.

Factors of 14	Sum of Factors
–1 and –14	–15
–2 and –7	–9

The factored form of $x^2 - 9x + 14$ is $(x - 2)(x - 7)$.

Practice & Problem Solving

Complete the table to factor the trinomial

39. $x^2 + 7x - 18$

Factors of c	Sum of Factors
■	■
■	■
■	7

Write the factored form of each trinomial.

40. $x^2 + 12x + 32$

41. $x^2 + 3x - 28$

42. $x^2 - 13x - 48$

43. $x^2 + 18xy + 45y^2$

44. **Look for Relationships** How are the binomial factors of $x^2 + 4x - 21$ and $x^2 - 4x - 21$ similar? How are they different?

Quick Review

To factor a trinomial of the form $ax^2 + bx + c$, find the factor pair of ac that has a sum of b. Then use the factors you found to write the binomials that have a product equal to the trinomial.

Example

What is the factored form of $2x^2 + 9x - 5$?

For the trinomial $2x^2 + 9x - 5$, $a = 2$ and $c = -5$, so $ac = -10$. Find the factor pair of -10 that has a sum of 9.

Factors of −10	Sum of Factors
−2 and 5	3
2 and −5	−3
−1 and 10	9

Since -1 and 10 are the correct factor pair, rewrite $9x$ as $-1x$ and $10x$.

$2x^2 + 9x - 5$

$= 2x^2 + 10x - 1x - 5$ ..... Rewrite.

$= (2x^2 + 10x) + (-1x - 5)$ ..... Group as two binomials.

$= 2x(x + 5) - 1(x + 5)$ ..... Factor out the GCFs.

$= (2x - 1)(x + 5)$ ..... Distributive Property

The factored form of $2x^2 + 9x - 5$ is $(2x - 1)(x + 5)$.

Practice & Problem Solving

Identify all of the factor pairs of ac you could use to rewrite b in order to factor each trinomial by grouping.

45. $5x^2 + 9x + 4$

46. $2x^2 + x - 15$

Write the factored form of each trinomial.

47. $3x^2 + 10x + 8$

48. $4x^2 - 3x - 10$

49. $5x^2 + 7x - 6$

50. $6x^2 + 13x + 6$

51. $10x^2 + 3x - 4$

52. $12x^2 + 22x + 6$

53. **Make Sense and Persevere** What are all the possible values of b for which $3x^2 + bx - 8$ is factorable using only integer coefficients and constants?

54. **Reason** A parking lot has an area of $6x^2 + 27x - 15$ square meters. Use factoring to find possible dimensions of the parking lot. The parking lot is to be enlarged so that each dimension is 5 meters greater than it was originally. What are the new dimensions of the parking lot? What is the new area of the parking lot?

LESSON 7-7 Factoring Special Cases

Quick Review

A **perfect-square trinomial** results when a binomial is squared.

Factor a perfect-square trinomial:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Use these patterns when the first and last terms are perfect squares and the middle term is twice the product of the numbers being squared.

Factor a difference of two squares:

$a^2 - b^2 = (a + b)(a - b)$

Use this pattern when a binomial can be written as a difference of two squares.

Example

What is the factored form of $9x^2 - 121$?

Write the first and last term as a perfect square.

$9x^2 - 121 = (3x)^2 - 11^2$

$= (3x - 11)(3x + 11)$

Practice & Problem Solving

Identify the value of c that would make each trinomial factorable using the perfect-square pattern.

55. $x^2 + 16x + c$

56. $2x^2 - 28x + c$

Write the factored form of each expression.

57. $x^2 + 10x + 25$

58. $x^2 - 121$

59. $x^2 - 18x + 81$

60. $9x^2 - 49y^2$

61. $3x^2 + 18x + 27$

62. $4x^2 - 56x + 196$

63. **Reason** Is the expression $3x^2 - 49$ factorable using only integer coefficients and constants? Explain why or why not.

64. **Make Sense and Persevere** The area of a playground is $36x^2 - 16y^2$ square feet. Use factoring to find possible dimensions of the playground. How are the side lengths related? What value would you need to subtract from the longer side and add to the shorter side for the playground to be a square?