TOPIC 8

Topic Review

TOPIC ESSENTIAL QUESTION

1. How can you use sketches and equations of quadratic functions to model situations and make predictions?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The graph of a quadratic function is a(n) _____.
3. The function $f(x) = x^2$ is called the _____.
4. To model the height of an object launched into the air t seconds after it is launched, you can use the _____.
5. The _____ is $f(x) = ax^2 + bx + c$.
6. A(n) _____ is a method used to find a quadratic function that best fits a data set.

- parabola
- quadratic parent function
- quadratic regression
- standard form of a quadratic function
- vertex form of a quadratic function
- vertical motion model

Concepts & Skills Review

LESSON 8-1 Key Features of Quadratic Functions

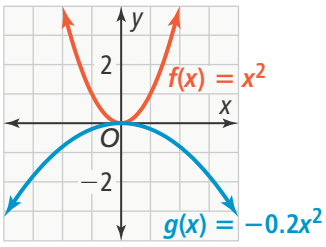
Quick Review

The graph of $f(x) = ax^2$ is a **parabola** with **vertex** (0, 0) and **axis of symmetry** $x = 0$. When $a > 0$, the parabola opens upward and the function has a minimum at the vertex. When $a < 0$, the parabola opens downward and the function has a maximum at the vertex.

Example

Compare the graph of $g(x) = -0.2x^2$ with the graph of $f(x) = x^2$.

The graph of g opens downward and is wider than the graph of f. For both graphs, the axis of symmetry is $x = 0$ and the vertex is (0, 0).



Practice and Problem Solving

Compare the graph of each function with the graph of $f(x) = x^2$.

7. $g(x) = 1.5x^2$
8. $h(x) = -9x^2$
9. **Communicate Precisely** Explain how you can you tell whether a function of the form $f(x) = ax^2$ has a minimum or a maximum value and what that value is.
10. **Model With Mathematics** Artificial turf costs \$15/sq ft to install, and sod costs \$0.15/sq ft to install. Write a quadratic function that represents the cost of installing artificial turf on a square plot with a side length of x feet, and a second quadratic function that represents the cost of installing sod on the same plot. How do the graphs of the two functions differ?

LESSON 8-2 Quadratic Functions in Vertex Form

Quick Review

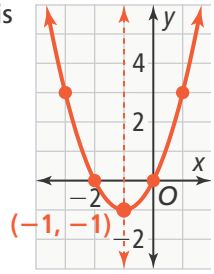
The **vertex form of a quadratic function** is $f(x) = a(x - h)^2 + k$. The vertex of the graph is at (h, k) and the axis of symmetry is $x = h$.

Example

Graph the function $f(x) = (x + 1)^2 - 1$.

The vertex is $(-1, -1)$ and the axis of symmetry is $x = -1$.

Use the points (0, 0) and (1, 3) to find two other points. Reflect each point across the axis of symmetry.



Practice and Problem Solving

11. Look for Relationships Graph the functions below. How are the graphs alike? How are the graphs different from each other?

$f(x) = -5(x - 3)^2 + 2$

$g(x) = -2(x - 3)^2 + 2$

Identify the vertex and axis of symmetry of the graph of each function.

12. $g(x) = (x + 8)^2 + 1$ **13.** $h(x) = (x - 5)^2 - 2$

14. An astronaut on the moon throws a moon rock into the air. The rock's height, in meters, above the moon's surface x seconds after it is thrown can be determined by the function $h(x) = -1.6(x - 2.5)^2 + 15$. What is the maximum height of the rock above the moon's surface? How many seconds after being thrown does the rock reach this height?

LESSON 8-3 Quadratic Functions in Standard Form

Quick Review

The standard form of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$. The y-intercept is c and the axis of symmetry, which is also the x-coordinate of the vertex, is $x = -\frac{b}{2a}$.

Example

Graph the function $f(x) = 3x^2 - 6x + 2$.

The y-intercept is 2.

Find the axis of symmetry.

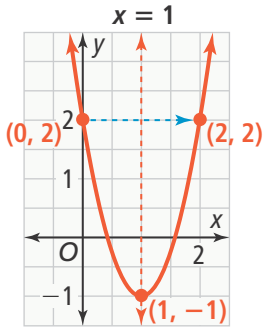
$$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$$

Find the y-coordinate of the vertex.

$f(1) = 3(1)^2 - 6(1) + 2 = -1$

Plot the vertex (1, −1) and identify the axis of symmetry.

Plot the y-intercept (0, 2). Reflect that point across the axis of symmetry.



Practice and Problem Solving

Identify the y-intercept, axis of symmetry, and vertex of the graph of each function.

15. $g(x) = -x^2 + 4x + 5$ **16.** $h(x) = -3x^2 + 7x + 1$

17. When given a function in standard form, how can you determine if the parabola has a minimum or maximum value?

18. Graph the function $f(x) = -3x^2 + 12x + 5$.

19. Reason A ball is tossed into the air. The function $f(x) = -16x^2 + 4x + 5$ represents the height in feet of the ball x seconds after it is thrown. At what height was the ball tossed into the air?

LESSON 8-4 Modeling With Quadratic Functions

Quick Review

Quadratic functions can model situations. For example, the vertical motion model is a quadratic function.

Example

Alberto launches an emergency flare at an initial velocity of 64 ft/s from an initial height of 6 ft. The flare must reach a height of 100 ft to be seen by a rescue team. Is Alberto's launch successful?

Substitute 64 for v_0 and 6 for h_0 in the vertical motion model.

$$h(t) = -16t^2 + 64t + 6$$

Find the vertex $(t, h(t))$.

$$t = -\frac{b}{2a} = -\frac{64}{2(-16)} = 2$$

$$h(2) = -16(2)^2 + 64(2) + 6 = 70$$

The vertex is (2, 70).

The flare will reach a maximum height of 70 ft, so Alberto's launch is not successful.

Practice and Problem Solving

Write a function h to model the vertical motion for each situation, given $h(t) = -16t^2 + v_0t + h_0$. Find the maximum height.

20. initial velocity: 54 ft/s
initial height: 7 ft

21. initial velocity: 18 ft/s
initial height: 9 ft

Write a quadratic function to represent the area of each rectangle. Graph the function. Interpret the vertex and intercepts. Identify a reasonable domain and range.

22.

23.

24. Make Sense and Persevere Given a vertical motion model, how can you identify the amount of time an object is in the air before it reaches the ground?

LESSON 8-5 Linear, Exponential, and Quadratic Models

Quick Review

To determine which function best models a data set, analyze the differences and ratios between consecutive y-values when the differences in consecutive x-values are constant.

Example

Determine whether the function below is linear, quadratic, or exponential.

x	y	1st Diff.	2nd Diff.	Ratios
0	1			
1	3	2		3
2	9	6	4	3
3	27	18	12	3

Since the ratio between the y-values is constant, the function is exponential.

Practice and Problem Solving

25. Make Sense and Persevere What is the first step in determining whether a table shows a linear, quadratic, or exponential function?

Determine whether the data in the tables represent a linear, quadratic, or exponential function.

26.

x	0	1	2	3	4
y	3	7	19	39	67

27.

x	−2	0	2	4	6
y	−20	−6	8	22	36