TOPIC 9

Topic Review

TOPIC ESSENTIAL QUESTION

1. How do you use quadratic equations to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The __________ is $ax^2 + bx + c = 0$, where $a \neq 0$.
3. The process of adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$ to form a perfect-square trinomial is called __________.
4. The x-intercepts of the graph of the function are also called the __________.
5. The __________ states that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where both a and b are greater than or equal to 0.
6. The __________ states that for all real numbers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$.

- completing the square
- discriminant
- Product Property of Square Roots
- quadratic equation
- quadratic formula
- standard form of a quadratic equation
- Zero-Product Property
- zeros of a function

Concepts & Skills Review

LESSON 9-1 Solving Quadratic Equations Using Graphs and Tables

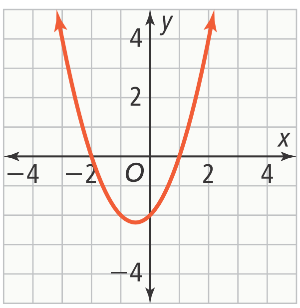
Quick Review

A **quadratic equation** is an equation of the second degree. A quadratic equation can have 0, 1 or 2 solutions, which are known as the **zeros of the related function**.

Example

Find the solutions of $0 = x^2 + x - 2$.

The x-intercepts of the related function are −2, and 1, so the equation has two real solutions.



From the graph, the solutions of the equation $x^2 + x - 2 = 0$ appear to be $x = -2$ and $x = 1$.

It is important to verify those solutions by substituting into the equation.

$(-2)^2 + (-2) - 2 = 0$ $\quad$ $1^2 + 1 - 2 = 0$

$0 = 0$ $\quad$ $0 = 0$

Practice & Problem Solving

Solve each quadratic equation by graphing.

7. $x^2 - 16 = 0$
8. $x^2 - 6x + 9 = 0$
9. $x^2 + 2x + 8 = 0$
10. $2x^2 - 11x + 5 = 0$

Find the solutions for each equation using a table. Round to the nearest tenth.

11. $x^2 - 64 = 0$
12. $x^2 - 6x - 16 = 0$
13. **Model With Mathematics** A video game company uses the profit model $P(x) = -x^2 + 14x - 39$, where x is the number of video games sold, in thousands, and $P(x)$ is the profit earned in millions of dollars. How many video games would the company have to sell to earn a maximum profit? How many video games would the company have to sell to not show a profit?

TOPIC 9 REVIEW

LESSON 9-2 Solving Quadratic Equations by Factoring

Quick Review

The **standard form of a quadratic equation** is $ax^2 + bx + c = 0$, where $a \neq 0$. The **Zero-Product Property** states that for all real numbers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$. The solutions of a quadratic equation can often be determined by factoring.

Example

How can you use factoring to solve $x^2 + 4x = 12$?

First write the equation in standard form.

$x^2 + 4x - 12 = 0$

Then, rewrite the standard form of the equation in factored form.

$(x - 2)(x + 6) = 0$

Use the Zero-Product Property. Set each factor equal to zero and solve.

$x - 2 = 0$ or $x + 6 = 0$

$x = 2$ $\quad$ $x = -6$

The solutions of $x^2 + 4x - 12 = 0$ are $x = 2$ and $x = -6$.

Practice & Problem Solving

Solve each equation by factoring.

14. $x^2 + 6x + 9 = 0$

15. $x^2 - 3x - 10 = 0$

16. $x^2 - 12x = 0$

17. $2x^2 - 7x - 15 = 0$

Factor, find the coordinates of the vertex of the related function, and then graph it.

18. $x^2 - 12x + 20 = 0$

19. $x^2 - 8x + 15 = 0$

20. Error Analysis Describe and correct the error a student made in factoring.

$$2x^2 - 8x + 8 = 0$$
$$2(x^2 - 4x + 4) = 0$$
$$2(x - 2)(x - 2) = 0$$
$$x = -2$$

LESSON 9-3 Rewriting Radical Expressions

Quick Review

A radical expression in simplest form has no perfect square factors other than 1 in the radicand. The **Product Property of Square Roots** states that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, when $a \geq 0$ and $b \geq 0$.

Example

Write an expression for $5\sqrt{3x} \cdot 2\sqrt{12x^3}$ without any perfect squares in the radicand.

$5\sqrt{3x} \cdot 2\sqrt{12x^3}$ ……… Multiply the constants, and use the Product Property of Square Roots to multiply the radicands.

$= 5 \cdot 2\sqrt{3x \cdot 12x^3}$

$= 10\sqrt{36x^4}$ ……… Simplify.

$= 10 \cdot 6 \cdot x^2$ ……… Simplify.

$= 60x^2$

The expression $5\sqrt{3x} \cdot 2\sqrt{12x^3}$ is equivalent to $60x^2$.

Practice & Problem Solving

Write an equivalent expression without a perfect square factor in the radicand.

21. $\sqrt{420}$

22. $4\sqrt{84}$

23. $\sqrt{35x} \cdot \sqrt{21x}$

24. $\sqrt{32x^5} \cdot \sqrt{24x^7}$

Compare each pair of radical expressions.

25. $2x^2\sqrt{21x}$ and $\sqrt{84x^5}$

26. $3xy\sqrt{15xy^2}$ and $\sqrt{135x^4y^3}$

27. Model With Mathematics A person's walking speed in inches per second can be approximated using the expression $\sqrt{384\ell}$, where ℓ is the length of a person's leg in inches. Write the expression in simplified form. What is the walking speed of a person with a leg length of 31 in.

LESSON 9-4 Solving Quadratic Equations Using Square Roots

Quick Review

To solve a quadratic equation using square roots, isolate the variable and find the square root of both sides of the equation.

Example

Use the properties of equality to solve the quadratic equation $4x^2 - 7 = 57$.

Rewrite the equation in the form $x^2 = a$.

$4x^2 - 7 = 57$

$4x^2 = 64$ Rewrite using the form $x^2 = a$, where a is a real number.

$x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$ Take the square root of each side of the equation.

$x = \pm 4$

Since 16 is perfect square, there are two integer answers. The solutions of the quadratic equation $4x^2 - 7 = 57$ are $x = -4$ and $x = 4$.

Practice & Problem Solving

Solve each equation by inspection.

28. $x^2 = 289$

29. $x^2 = -36$

30. $x^2 = 155$

31. $x^2 = 0.64$

Solve each equation.

32. $5x^2 = 320$

33. $x^2 - 42 = 358$

34. $4x^2 - 18 = 82$

35. Higher Order Thinking Solve $(x - 4)^2 - 81 = 0$. Explain the steps in your solution.

36. Communicate Precisely Use the equation $d = \sqrt{(12 - 5)^2 + (8 - 3)^2}$ to calculate the distance between the points (3, 5) and (8, 12). What is the distance?

LESSON 9-5 Completing the Square

Quick Review

The process of adding $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$ to form a perfect-square trinomial is called **completing the square**. This is useful for changing $ax^2 + bx + c$ to the form $a(x - h)^2 + k$.

Example

Find the solutions of $x^2 - 16x + 12 = 0$.

First, write the equation in the form $ax^2 + bx = d$.

$x^2 - 16x = -12$

Complete the square.

$b = -16$, so $\left(\frac{-16}{2}\right)^2 = 64$

$x^2 - 16x + 64 = -12 + 64$

$x^2 - 16x + 64 = 52$

Write the trinomial as a binomial squared.

$(x - 8)^2 = 52$

Solve for x.

$x - 8 = \sqrt{52}$

$x = 8 \pm 2\sqrt{13}$

$x = 8 + 2\sqrt{13}$ and $x = 8 - 2\sqrt{13}$.

Practice & Problem Solving

Find the value of c that makes each expression a perfect-square trinomial. Then write the expression as a binomial squared.

37. $x^2 + 18x + c$

38. $x^2 - 6x + c$

39. $x^2 - 15x + c$

40. $x^2 + 24x + c$

Solve each equation by completing the square.

41. $x^2 + 18x = 24$

42. $x^2 - 10x = 46$

43. $x^2 + 22x = -39$

44. $3x^2 + 42x + 45 = 0$

45. Construct Arguments To solve the equation $x^2 - 9x - 15 = 0$, would you use graphing, factoring, or completing the square if you want exact solutions? Explain.

Quick Review

The **quadratic formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, gives solutions of quadratic equations in the form $ax^2 + bx + c = 0$ for real values of a, b, and c where $a \neq 0$. The quadratic formula is a useful method to find the solutions of quadratic equations that are not factorable.

The **discriminant** is the expression $b^2 - 4ac$, which indicates the number of solutions of the equation. The solutions of a quadratic equation are also called its **roots**, which are the input values when the related function's output value is zero.

If $b^2 - 4ac > 0$, there are 2 real solutions.

If $b^2 - 4ac = 0$, there is 1 real solution.

If $b^2 - 4ac < 0$, there are no real solutions.

Example

Use the quadratic formula to find the solutions of $x^2 - 9 = 5x$.

Write the equation in standard form $ax^2 + bx + c = 0$ and identify a, b and c.

$x^2 - 5x - 9 = 0$

$a = 1$, $b = -5$, $c = -9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{(-5)^2 - 4(1)(-9)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{61}}{2}$$

$$x = \frac{5 + \sqrt{61}}{2} \approx 6.41 \text{ and}$$

$$= \frac{5 - \sqrt{61}}{2} \approx -1.41$$

The approximate solutions of $x^2 - 9 = 5x$ are $x \approx 6.41$ and $x \approx -1.41$.

Practice & Problem Solving

Solve each equation using the quadratic formula.

46. $2x^2 + 3x - 5 = 0$

47. $-5x^2 + 4x + 12 = 0$

48. $3x^2 + 6x - 1 = 4$

49. $4x^2 + 12x + 6 = 0$

Use the discriminant to determine the number of real solutions for each equation.

50. $3x^2 - 8x + 2 = 0$

51. $-4x^2 - 6x - 1 = 0$

52. $7x^2 + 14x + 7 = 0$

53. $2x^2 + 5x + 3 = -5$

54. **Error Analysis** Describe and correct the error a student made in solving $3x^2 - 5x - 8 = 0$.

$a = 3$, $b = -5$, $c = 8$

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4(3)(8)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{-71}}{6}$$

There are no real solutions.

55. **Reason** The function $f(x) = -5x^2 + 20x + 55$ models the height of a ball x seconds after it is thrown into the air. What are the possible values of the discriminant of the related equation? Explain.

LESSON 9-7 Solving Systems of Linear and Quadratic Equations

Quick Review

A **linear-quadratic system** of equations includes a linear equation and a quadratic equation. The graph of the system of equations is a line and a parabola.

$y = mx + b$

$y = ax^2 + bx + c$

You can solve a linear-quadratic system of equations by graphing, elimination, or substitution.

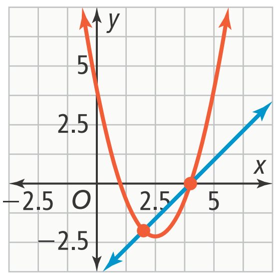
Example

What are the solutions of the system of equations?

$\boldsymbol{y = x^2 - 5x + 4}$

$\boldsymbol{y = x - 4}$

Graph the equations in the system on the same coordinate plane.



The solutions are where the parabola and the line intersect, which appear be at the points (2, −2) and (4, 0).

Check that the ordered pairs are solutions of the equations $y = x^2 - 5x + 4$ and $y = x - 4$.

$-2 = (2)^2 - 5(2) + 4$

$-2 = 4 - 10 + 4$

$-2 = -2$

and

$0 = (4)^2 - 5(4) + 4$

$0 = 16 - 20 + 4$

$0 = 0$

$0 = 4 - 4$

$0 = 0$

and

$-2 = 2 - 4$

$-2 = -2$

The solutions of the system are (2, −2) and (4, 0).

Practice & Problem Solving

Rewrite each equation as a system of equations, and then use a graph to solve.

56. $4x^2 = 2x - 5$

57. $2x^2 + 3x = 2x + 1$

58. $x^2 - 6x = 2x - 16$

59. $0.5x^2 + 4x = -12 - 1.5x$

Find the solution(s) of each system of equations.

60. $y = x^2 + 6x + 9$
$y = 3x$

61. $y = x^2 + 8x + 30$
$y = 5 - 2x$

62. $y = 3x^2 + 2x + 1$
$y = 2x + 1$

63. $y = 2x^2 + 5x - 30$
$y = 2x + 5$

64. **Make Sense and Persevere** Write an equation for a line that does not intersect the graph of the equation $y = x^2 + 6x + 9$.

65. **Reason** A theater company uses the revenue function $R(x) = -50x^2 + 250x$, where x is the ticket price in dollars. The cost function of the production is $C(x) = 450 - 50x$. What ticket price is needed for the theater to break even?