

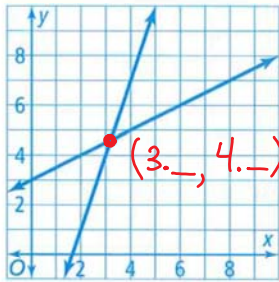
1-6

Linear Systems

PearsonRealize.com

EXPLORE & REASON

The graph shows two lines that intersect at one point.



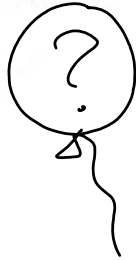
A. What are the approximate coordinates of the point of intersection?

Guess: $(3.2, 4.7)$

B. How could you verify whether the coordinates you estimated are, in fact, the solution? Is the point the solution to the equations of both lines?

Intersection
→ true for both lines

C. **Make Sense and Persevere** Use your result to refine your approximation, and try again. Can you find the point of intersection this way? Is there a more efficient way? © MP.1

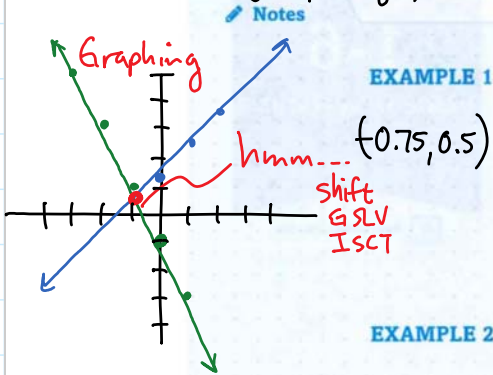


HABITS OF MIND

Communicate Precisely The graphs of two equations appear to intersect at the point $(2, 3)$. Does that guarantee that $x = 2$ and $y = 3$ is a solution to both equations? Explain. © MP.6

No!

Systems of Linear Eqns:
 → graphing, substitution, elimination



$$* -16x = 12$$

$$x = \frac{12}{-16} = -\frac{6}{8} = -\frac{3}{4}$$

EXAMPLE 1

Try It! Solve a System of Linear Equations

1. Solve each system of equations

a. $\begin{cases} 2x + y = -1 \\ 5y - 6x = 7 \end{cases}$

b. $\begin{cases} 3x + 2y = 5 \\ 6x + 4y = 3 \end{cases}$

or
Subst.

$$y = -2x - 1$$

$$y = \frac{6}{5}x + \frac{7}{5}$$

$$-2x - 1 = \frac{6}{5}x + \frac{7}{5}$$

$$-10x - 5 = 6x + 7$$

Elimination: add opps or Subt same
 • LCM
 $6x + 4y = 3$
 $6x + 4y = -10$
 $0 = -7$
 Variables disappear
 FALSE
 → No Solution...
 • parallel lines...

If TRUE, then same line → ∞ solutions

EXAMPLE 2

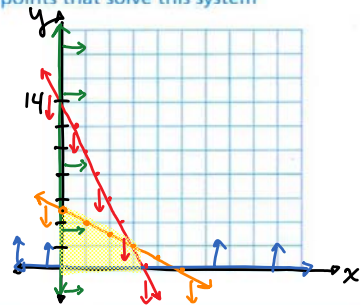
Try It! Solve a System of Linear Inequalities

2. Sketch the graph of the set of all points that solve this system of linear inequalities.

$$\begin{cases} 2x + y \leq 14 \\ x + 2y \leq 10 \end{cases}$$

* $x \geq 0$ vert
 * $y \geq 0$ horiz

border
 \leq, \geq solid
 $<, >$ dash



HABITS OF MIND

Make Sense and Persevere Is it possible to solve a system of linear inequalities using the same methods you used to solve a system of linear equations? © MP.1

EXAMPLE 3

Try It! Solve a System of Equations in Three Variables

3. Solve the following systems of equations.

a. $\begin{cases} x + y + z = 3 \\ x - y + z = 1 \\ x + y - z = 2 \end{cases}$

b. $\begin{cases} 2x + y - 2z = 3 \\ x - 2y + 7z = 12 \\ 3x - y + 5z = 10 \end{cases}$

GC → Equa → Simultaneous

... Ma Error
 → no solution

HABITS OF MIND

Generalize What is the goal of the substitution and elimination methods? © MP.8

• eliminate variables & solve ...

Elimination
 • add opps or subt "same"

ex 3a) $\begin{cases} x + y + z = 3 \\ x - y + z = 1 \end{cases}$
 (+) $2x + 2z = 4$
 $x + z = 2$

$\frac{3}{2} + z = 2$
 $z = 2 - \frac{3}{2}$
 $z = \frac{1}{2}$

$2x = 3$
 $x = \frac{3}{2}$

$\frac{3}{2} + y - \frac{1}{2} = 2$
 $y + 1 = 2$
 $y = 1$

EXAMPLE 4 Try It! Write a System of Equations as a Matrix

4. Write the matrix for the system of equations or the system of equations for the matrix.

a.
$$\begin{cases} 3x - y = 4 \\ -2x + 7y = 20 \end{cases}$$

$$\rightarrow \left[\begin{array}{cc|c} 3 & -1 & 4 \\ -2 & 7 & 20 \end{array} \right]$$

b.
$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 4 \\ 8 & -1 & -2 & 5 \\ 2 & 0 & 1 & 9 \end{array} \right]$$

augmented matrix 3×4

Coefficient matrix 3×3

$$\left[\begin{array}{ccc} 0 & 2 & 3 \\ 8 & -1 & -2 \\ 2 & 0 & 1 \end{array} \right]$$

Constant matrix 3×1

$$\left[\begin{array}{c} 4 \\ 5 \\ 9 \end{array} \right]$$

$$\rightarrow \begin{cases} 2y + 3z = 4 \\ 8x - y - 2z = 5 \\ 2x + z = 9 \end{cases}$$

EXAMPLE 5 Try It! Relate Systems of Equations and Matrices

5. a. Write the system of equations described by the augmented matrix. Describe a real-world situation that could be modeled by the system.

$$\left[\begin{array}{cc|c} 1 & 1 & 10 \\ 3 & 2 & 80 \end{array} \right]$$

b. What would the matrix $\left[\begin{array}{cc|c} 1 & 0 & 20 \\ 0 & 1 & 10 \end{array} \right]$ represent in terms of your real-world situation?

HABITS OF MIND

Communicate Precisely What characteristics must a system of equations have for it to be appropriate to rewrite it in matrix form? © MP.6

Do You UNDERSTAND?

- ESSENTIAL QUESTION** How can you find and represent solutions of systems of linear equations and inequalities?

- Error Analysis** Shandra said the solution of the system of equations $\begin{cases} 2x + y = 3 \\ -x + 4y = -6 \end{cases}$ is $(-1, 2)$. Is she correct? Explain. © MP.3

- Communicate Precisely** Why is a system of linear inequalities often solved graphically? © MP.6

- Make Sense and Persevere** How does knowing how to solve a system of two equations in two variables help you to solve a system of three equations in three variables? © MP.1

- Vocabulary** What is the difference between a coefficient matrix and an augmented matrix?

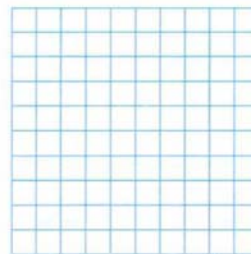
Do You KNOW HOW?

- Solve the following system of equations.

$$\begin{cases} 2x + 2y = 10 \\ x + 5y = 13 \end{cases}$$

- Graph the following system of inequalities.

$$\begin{cases} -x + 2y < 1 \\ x \geq 0 \\ y \geq 0 \end{cases}$$



- Write the system of equations represented by the matrix $\begin{bmatrix} 1 & -2 & 2 \\ -4 & 3 & -5 \end{bmatrix}$.

- Equations with two variables that are raised only to the first power represent lines. There are three possible outcomes for the intersections of two lines. Describe the outcomes.