Activity

10-1
Operations With Matrices

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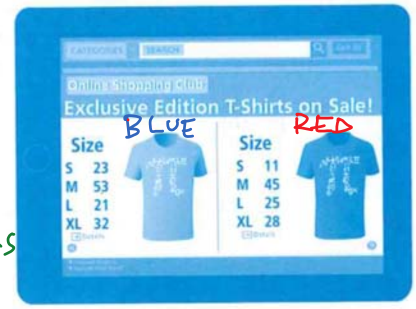
MODEL & DISCUSS

This screen shows the number of Small, Medium, Large, and Extra Large limited-edition silkscreen shirts on sale at an online boutique.



rows & columns

A. Construct a table to summarize the inventory that is on sale.

$$\begin{array}{r c} & \begin{array}{cccc} S & M & L & XL \end{array} \\ \text{Blue T} \rightarrow & \left[\begin{array}{cccc} 23 & 53 & 21 & 32 \\ 11 & 45 & 25 & 28 \end{array}\right] \\ \text{Red T} \rightarrow & \end{array}$$

B. At the end of the day, the boutique has sold this many of each T-shirt from the sale items: red: 4 S, 6 M, 3 L, 5 XL; blue: 2 S, 8 M, 4 L, 0 XL. Make two new tables, one showing the merchandise sold and one showing the inventory that is left.

SOLD

$$\begin{bmatrix} 2 & 8 & 4 & 0 \\ 4 & 6 & 3 & 5 \end{bmatrix}$$

LEFT = INVENTORY − SOLD

$$\begin{bmatrix} 21 & 45 & 17 & 32 \\ 7 & 39 & 22 & 23 \end{bmatrix}$$

C. Use Structure What relationships did you use in creating the two tables in Part B? MP.7

rows : Red & Blue T-shirts

Columns: T-shirt Sizes

left: Inventory − sold

HABITS OF MIND

Make Sense and Persevere Edwin summarized the information given in the problem in 2 rows and 4 columns. In Part B, he summarized the information about the number of shirts sold in a table that had 4 rows and 2 columns. Was this organizational strategy helpful? Explain. MP.1

Swapping rows & columns → messy

Matrix – table of data values (elements) using rows & columns

$$\begin{bmatrix} \text{rows} \rightarrow & \\ \text{columns} \downarrow & \end{bmatrix}$$

elements

Notes

address: a_{mn} Subscript

m: row #

n: Column #

$m \times n$: dimensions

EXAMPLE 1

Try It! Represent Data With a Matrix

1. In matrix C, the entries are the numbers of students on a committee. Column 1 lists girls, column 2 lists boys, row 1 lists sophomores, and row 2 lists juniors. Find a_{12}, a_{21}, and a_{22}, and tell what each number represents.

name •Capitalized

$$\begin{array}{ccc} & G & B \\ C = & \begin{bmatrix} 7 & 5 \\ 8 & 10 \end{bmatrix} & \begin{array}{c} S \\ J \end{array} \end{array}$$

$a_{12} = 5 \rightarrow$ 5 sophomore boys

row#

Column#

$a_{21} = 8$ junior girls

$a_{22} = 10$ junior boys

Equal Matrices

- same dimension
- each element corresponds to each other's address

book ex 1c) $\begin{bmatrix} 12 & 0 \\ 2x & 10 \end{bmatrix} = \begin{bmatrix} 12 & y \\ 14 & 10 \end{bmatrix}$

a_{21} a_{12}

what are x & y ?

x : row 2 colm 1 $\frac{2x}{2} = \frac{14}{2}$ $x = 7$

y : $0 = y$

Scalar multiplication

- multiplication of each element by a single real number, called a scalar.

EXAMPLE 2

Try It! Apply Scalar Multiplication

2. In this matrix C, the rows represent prices for shirts and khakis. The columns have the same meaning as in Example 2. If the sales tax rate is 6%, use scalar multiplication to find the sales tax for each item.

.06

$C = \begin{bmatrix} 75 & 40 & 25 \\ 100 & 60 & 30 \end{bmatrix} \rightarrow$

$.06C = .06\begin{bmatrix} 75 & 40 & 25 \\ 100 & 60 & 30 \end{bmatrix} = \begin{bmatrix} 4.5 & 2.4 & 1.5 \\ 6 & 3.6 & 1.8 \end{bmatrix}$

2×3

HABITS OF MIND

Generalize Let the dimensions of matrix Z be 3 × 4. After multiplying this matrix by a scalar, what are the dimensions of the product matrix? Explain. MP.8

↳ will not change

Notes

→ same dimensions ...

Assess

EXAMPLE 3 **Try It!** Add and Subtract Matrices

3. Consider matrices M and N.

$M = \begin{bmatrix} -3 & 5 \\ 2 & 0 \end{bmatrix}$, $N = \begin{bmatrix} 6 & 5 \\ -8 & 0.2 \end{bmatrix}$

a. What are matrices $M + N$ and $N + M$?

same for both

$\begin{bmatrix} -3+6 & 5+5 \\ 2+-8 & 0+0.2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 10 \\ -6 & 0.2 \end{bmatrix}$

b. What are matrices $M - N$ and $N - M$?

$M-N = \begin{bmatrix} -9 & 0 \\ 10 & -0.2 \end{bmatrix}$ ∫∫ $N-M = \begin{bmatrix} 9 & 0 \\ -10 & 0.2 \end{bmatrix}$

opposite

$M-N = N-M$
$M-N = -(-N+M)$
$M-N = -(M-N)$

additive inverse

EXAMPLE 4 **Try It!** Understand Matrix Addition and Subtraction

4. Consider the matrices below.

$P = \begin{bmatrix} 5 & 2 & -3 \\ 7 & 0 & -5 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & -2 \\ 5 & -5 \\ -7 & 7 \end{bmatrix}$, $R = \begin{bmatrix} 6 & 0.5 \\ -3 & 0 \\ -2 & -2 \end{bmatrix}$

2x3 3x2 3x2

additive inverse:

zero matrix

$M + -(M) = 0$

a. Find $R - Q$. What other matrix sums or differences can be calculated?

$R-Q = \begin{bmatrix} 4 & 2.5 \\ -8 & 5 \\ 5 & -9 \end{bmatrix}$ / $Q-R$, $Q+R$, $R+Q$, $P+P$, $P-P$, $Q+Q$, $Q-Q$, $R+R$, $R-R$

b. Find the additive inverses of P, Q, and R.

$-P = \begin{bmatrix} -5 & -2 & 3 \\ -7 & 0 & 5 \end{bmatrix}$ $-Q = \begin{bmatrix} -2 & 2 \\ -5 & 5 \\ 7 & -7 \end{bmatrix}$ $-R = \begin{bmatrix} -6 & -0.5 \\ 3 & 0 \\ 2 & 2 \end{bmatrix}$

HABITS OF MIND

Communicate Precisely What must be true about two matrices for their sum or difference to exist? MP.6

→ must have the same dimensions

EXAMPLE 5 **Try It!** Use Matrices to Translate and Dilate Figures

"instructions" → matrix

5. A segment has endpoints $M(8, -7)$ and $N(1, 2)$.

preimage

image

a. Use matrices to represent a translation of $\overline{MN}$ to $\overline{RS}$ by 6 units left and 3 units down. What are the coordinates of R and S?

y-value

x-value

Point M: $\begin{bmatrix} 8 \\ -7 \end{bmatrix}$ Point N: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ endpts of $\overline{MN}$: $\begin{bmatrix} 8 & 1 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & -6 \\ -3 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & -5 \\ -10 & -1 \end{bmatrix}$ $\overline{RS}$

b. Use matrices to represent a dilation of $\overline{MN}$ to $\overline{DE}$ by a scale factor of 3, centered at the origin. What are the coordinates of D and E?

Scalar mult

Scalar

$3 \cdot \begin{bmatrix} 8 & 1 \\ -7 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 24 & 3 \\ -21 & 6 \end{bmatrix}$ $\overline{DE}$

HABITS OF MIND

Model With Mathematics The matrix $T = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \end{bmatrix}$ represents a triangle. Use matrices to determine whether dilating by a factor of 2 and then translating 5 units right is the same as translating the triangle 5 units right and then dilating by a factor of 2. Does the order of the transformations matter? Explain. MP.4

$2 \cdot \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \stackrel{?}{=} 2 \cdot \left(\begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 0 & 0 & 0 \end{bmatrix} \right)$

$\rightarrow \begin{bmatrix} 7 & 7 & 13 \\ 4 & 6 & 4 \end{bmatrix} \neq \begin{bmatrix} 12 & 12 & 18 \\ 4 & 6 & 4 \end{bmatrix}$

→ order of operations matters!!!

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you interpret matrices and operate with matrices?

2. **Error Analysis** Tonya says $\begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ would produce a zero matrix. Explain her error. MP.3

3. **Communicate Precisely** Explain how you know if two matrices can be added. Then explain how to add them. MP.6

4. **Vocabulary** What are equal matrices? Give an example of equal matrices.

Do You KNOW HOW?

Identify the element for each matrix.

5. $\begin{bmatrix} 4 & 1 & 0 \\ 7 & 3 & 5 \end{bmatrix}$; a_{23}

$a_{23} = 5$

6. $\begin{bmatrix} -6 \\ 2 \end{bmatrix}$; a_{11}

$a_{11} = -6$

Given $A = \begin{bmatrix} 3 & -2 \\ 7 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 7 \\ -4 & 12 \end{bmatrix}$, **calculate each of the following.**

7. $A + B$

$\begin{bmatrix} 3 & 5 \\ 3 & 13 \end{bmatrix}$

8. $B - A$

$\begin{bmatrix} -3 & 9 \\ -11 & 11 \end{bmatrix}$

opposites

9. $4A$

$4\begin{bmatrix} 3 & -2 \\ 7 & 1 \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ 28 & 4 \end{bmatrix}$

10. $A - B$

$\begin{bmatrix} 3 & -9 \\ 11 & -11 \end{bmatrix}$

11. The endpoints of $\overline{AB}$ are represented by the matrix $\begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix}$.
Find the image of the segment after a dilation, centered at the origin, by a scale factor of 2.

$2\begin{bmatrix} 3 & 7 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 14 \\ 2 & 10 \end{bmatrix}$