Activity

10-2 Matrix Multiplication

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EXPLORE & REASON

Two stores, Quick Repair and TechRite, buy and sell pre-owned phones, tablets, and computers. The matrices below represent their average revenue R, purchase costs C, and repair expenses E for each item:

revenue

R	Quick Repair	$150	$100	$400
	TechRite	$200	$250	$500

cost

C	Quick Repair	$100	$50	$200
	TechRite	$125	$75	$300

expenses

E	Quick Repair	$25	$20	$50
	TechRite	$10	$50	$50

A. Would it make sense to find the sum and/or difference of any two of the three matrices? Explain.

Matrices C & E ← — $$$

Cuts into revenue

→ not scalar multiplication (lower cased)

Matrices A & B

Find AB.

→ dimensions:

$A_{m \times n} \cdot B_{n \times r}$

Columns of Mat A = # rows of Mat B

... So $AB = C$ answer

... C's dimension is $m \times r$

B. Make Sense and Persevere Quick Repair and TechRite both need to estimate their total purchase and repair costs. They each predict that they will need to purchase 100 phones, 100 tablets, and 100 computers, and that they will need to repair 50% of them. Explain what you would do to find the total costs. MP.1

$C =$ [... Sum of the products ...]

row # of Mat A

Column # of Mat B

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ (2×3)

$A_{2\times3} \times B_{3\times2}$

$B = \begin{bmatrix} -9 & -8 \\ -7 & -6 \\ -5 & -4 \end{bmatrix}$ (3×2)

$C_{2\times2} = \begin{bmatrix} 1\cdot(-9)+2(-7)+3(-5) & 1(-8)+2(-6)+3(-4) \\ 4\cdot(-9)+5(-7)+6(-5) & 4(-8)+5(-6)+6(-4) \end{bmatrix} = \begin{bmatrix} -38 & -32 \\ -101 & -86 \end{bmatrix}$

(11, 12, 21, 22)

HABITS OF MIND

Reason Which store makes a greater profit from the sale of a repaired phone? Explain. MP.2

$D_{3\times2} = B_{3\times2} \times A_{2\times3}$

$= \begin{bmatrix} -9+-32 & -18+-40 & -27+-48 \\ -7+-24 & -14+-30 & -21+-36 \\ 5+-16 & -10+-20 & -15+-24 \end{bmatrix} = \begin{bmatrix} -41 & -58 & -75 \\ -31 & -44 & -57 \\ -11 & -30 & -39 \end{bmatrix}$

(11, 12, 13, 21, 22, 23, 31, 32, 33)

4

Notes

Assess

EXAMPLE 1 **Try It! Understand Matrix Multiplication**

1. How would the matrices need to change to determine GW instead of WG?

$\begin{bmatrix}7 & 0 & -2\end{bmatrix}_{1\times3}\cdot\begin{bmatrix}-7\\5\\-5\end{bmatrix}_{3\times1}=\begin{bmatrix}-49+0+10\end{bmatrix}_{1\times1}=[-39]$

$\begin{bmatrix}-7\\5\\-5\end{bmatrix}_{3\times1}\begin{bmatrix}7 & 0 & -2\end{bmatrix}_{1\times3}=\begin{bmatrix}\_ & \_ & \_\\ \_ & \_ & \_\\ \_ & \_ & \_\end{bmatrix}_{3\times3}$

Weighting

$W=\underset{1\times3}{\begin{bmatrix}\text{unit} & \text{proj} & \text{final}\\ 0.50 & 0.30 & 0.20\end{bmatrix}}$

Grades

$G=\underset{3\times2}{\begin{matrix}\text{unit}\\ \text{proj}\\ \text{final}\end{matrix}\begin{bmatrix}90 & 80\\ 95 & 70\\ 75 & 85\end{bmatrix}}$ (O ~ Oscar, R ~ Reagan)

Oscar: $0.50(90)+0.30(95)+0.20(75)$

Reagan: $0.50(86)+0.30(70)+0.20(85)$

$=\underset{1\times2}{[88.5 \quad 78]}$

EXAMPLE 2 **Try It! Examine Multiplication of Square Matrices**

2. Determine whether each equation may be true for the following matrices.

$A=\begin{bmatrix}3 & 0\\ -1 & -2\end{bmatrix}$, $B=\begin{bmatrix}-2 & 1\\ 3 & -4\end{bmatrix}$, $C=\begin{bmatrix}6 & 2\\ 4 & 8\end{bmatrix}$

a. $(AB)C=A(BC)$ (Assoc Prop of Mult)

b. $(A+B)C=AC+BC$ (Distributive Property on the left)

2c) $AB=BA$? (Commutative prop of mult)

$\begin{bmatrix}3 & 0\\ -1 & -2\end{bmatrix}\begin{bmatrix}-2 & 1\\ 3 & -4\end{bmatrix}\overset{?}{=}\begin{bmatrix}-2 & 1\\ 3 & -4\end{bmatrix}\begin{bmatrix}3 & 0\\ -1 & -2\end{bmatrix}$

$\begin{bmatrix}-6+0 & 3+0\\ 2-6 & -1+8\end{bmatrix}$ | $\begin{bmatrix}-6-1 & 0-2\\ 9+4 & 0+8\end{bmatrix}$

$\begin{bmatrix}-6 & 3\\ -4 & 7\end{bmatrix}\neq\begin{bmatrix}-7 & -2\\ 13 & 8\end{bmatrix}$

NO.....

$\begin{bmatrix}-6 & 3\\ -4 & 7\end{bmatrix}\cdot\begin{bmatrix}6 & 2\\ 4 & 8\end{bmatrix}\overset{?}{=}\begin{bmatrix}-2 & 1\\ 3 & -4\end{bmatrix}\begin{bmatrix}6 & 2\\ 4 & 8\end{bmatrix}$

$\begin{bmatrix}-36 & 12 & -12 & 24\\ -24 & 28 & -8 & 56\end{bmatrix}\overset{?}{=}\begin{bmatrix}-12+4 & -4+8\\ 18-16 & 6-32\end{bmatrix}$

$\begin{bmatrix}-24 & 12\\ 4 & 48\end{bmatrix}\overset{?}{=}\begin{bmatrix}-8 & 4\\ 2 & -26\end{bmatrix}$

$\begin{bmatrix}3 & 0\\ -1 & -2\end{bmatrix}\begin{bmatrix}-8 & 4\\ 2 & -26\end{bmatrix}$

$\begin{bmatrix}-24 & 12\\ 8-4 & -4+52\end{bmatrix}=\begin{bmatrix}-24 & 12\\ 4 & 48\end{bmatrix}$

Yes....

$\begin{bmatrix}1 & 1\\ 2 & -6\end{bmatrix}\begin{bmatrix}6 & 2\\ 4 & 8\end{bmatrix}=\begin{bmatrix}3 & 0\\ -1 & -2\end{bmatrix}\begin{bmatrix}6 & 2\\ 4 & 8\end{bmatrix}+\begin{bmatrix}-8 & 4\\ 2 & -26\end{bmatrix}$

$\begin{bmatrix}6 & 4 & 28\\ 12-24 & 4-48\end{bmatrix}\overset{?}{=}\begin{bmatrix}18 & 6 & 0\\ -6-8 & -2-16\end{bmatrix}+\begin{bmatrix}-8 & 4\\ 2 & -26\end{bmatrix}$

$\begin{bmatrix}10 & 10\\ -12 & -44\end{bmatrix}\overset{?}{=}\begin{bmatrix}18 & 6\\ -14 & -18\end{bmatrix}+\begin{bmatrix}-8 & 4\\ 2 & -26\end{bmatrix}$

$\begin{bmatrix}10 & 10\\ -12 & -44\end{bmatrix}$

Yes....

HABITS OF MIND

Construct Arguments Let $M=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ and $N=\begin{pmatrix}-2 & 1\\ 1.5 & -0.5\end{pmatrix}$. Is $MN=NM$? If so, can you conclude that matrix multiplication is commutative? MP.3

No...

Identity Matrix – Square matrix (# rows = # columns)

Notes

I

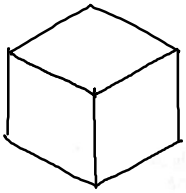
- square matrix of zeros w/ 1's along the downhill diagonal

$$\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

EXAMPLE 3 Try It! Understand Identity Matrices

3. a. What is the product of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$? Identity Matrix

$$\begin{bmatrix} a+0 & 0+b \\ c+0 & 0+d \end{bmatrix} \rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



$-I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

$I \cdot A$

b. What is the product of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$?

$$\begin{bmatrix} -1+0 & 0+0 \\ 0+0 & 0+-1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$I \cdot A = A$

$A \cdot I$?

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+0 & 0+0 \\ 0+0 & 0+-1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$:)

$I \cdot A = A \cdot I = A$

HABITS OF MIND

Use Structure Let $A = \begin{bmatrix} -1 & 0 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} j & k \\ l & m \end{bmatrix}$. $AB = \begin{bmatrix} -1 & 0 \\ 3 & 5 \end{bmatrix}$. What are the values of j, k, l, and m? MP.7

$$\begin{bmatrix} -1 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} j & k \\ l & m \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 3 & 5 \end{bmatrix}$$

$-1(j) + 0(l) = -1$

$-1(k) + 6(m) = 0$

$3(j) + 5(l) = 3$

$3(k) + 5(m) = 5$

etc.. use Subst..

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What does it mean to multiply a matrix by another matrix?

2. **Reason** Would it be possible to multiply $A_{3\times5}$ and $B_{4\times5}$? Explain your reasoning. MP.2

3. **Vocabulary** Explain why a matrix with ones on the main diagonal and zeros for all the other elements is called the *identity matrix*.

4. **Error Analysis** A student thought that the product of $A_{1\times5}$ and $B_{5\times1}$ should have five elements in the answer. Is the student correct? If not, how many elements will there be? MP.3

Do You KNOW HOW?

Let $A = \begin{bmatrix} 3 & 0 \\ -1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}$. AB ≠ BA

5. Find AB and BA to demonstrate that matrix multiplication is not commutative. Show your work.

$\begin{bmatrix} 3 & 0 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}$ vs. $\begin{bmatrix} -2 & 1 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & -2 \end{bmatrix}$

$\begin{bmatrix} -6 & 0 & 3 & 0 \\ 2 & -6 & -1 & 8 \end{bmatrix}$ $\begin{bmatrix} -6 & -1 & 0 & -2 \\ 9 & 4 & 0 & 8 \end{bmatrix}$

$\begin{bmatrix} -6 & 3 \\ -4 & 7 \end{bmatrix} \neq \begin{bmatrix} -7 & -2 \\ 13 & 8 \end{bmatrix}$

Find each product. Identity

6. $\begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 7. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 8 & 2 \end{bmatrix}$

AI = IA = A

→ $\begin{bmatrix} 4 & 7 \\ 1 & -2 \end{bmatrix}$ → $\begin{bmatrix} 5 & 0 \\ 8 & 2 \end{bmatrix}$

8. The coordinates of the vertices of a triangle are $A(-2, 3)$, $B(1, 1)$, and $C(2, -1)$. The coordinate are multiplied by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Find the coordinates of the image of the triangle after the transformation.

$\begin{bmatrix} -2 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ 2×3 2×2 3×2 or switch around

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 & 1 & 0 & 2 & 0 \\ 0 & -3 & 0 & -1 & 0 & 1 \end{bmatrix}$ (11, 12, 13, 21, 22, 23)

$= \begin{bmatrix} -2 & 1 & 2 \\ -3 & -1 & 1 \end{bmatrix}$ opp y values

reflection across x-axis (vert refl)