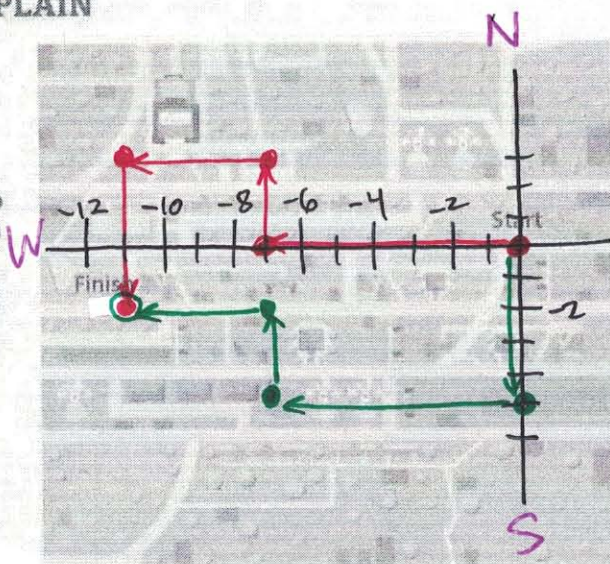


CRITIQUE & EXPLAIN

Olivia and Benito are taking part in a scavenger hunt. They are given a map that shows the start and finish line. They are also given a list of directions to the finish line. They get to choose how they want to follow the directions, so they took different paths.



* Olivia's path:	* Benito's path:
5 blocks south	7 blocks west
7 blocks west	3 blocks north
3 blocks north	4 blocks west
4 blocks west	5 blocks south

A. Will both Olivia and Benito reach the finish line? Explain.

Yes. → 11 blocks west & 2 blocks south

B. Create a different set of directions that would get someone to the finish line.

? 3 blocks north, 4 blocks west, 5 blocks south, 7 blocks west

C. Communicate Precisely Does the order of the instructions that pair distance and direction affect the outcome? Explain. © MP.6

No... not when distance & direction are paired.

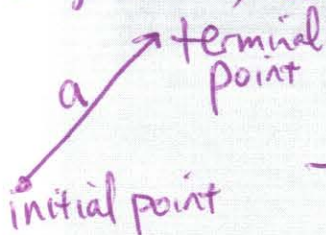
HABITS OF MIND

Look for Relationships Russel looked at his list of four directions and decided that he could reach the finish line in fewer blocks. What is a possible set of directions for a path he could take? © MP.7

Sample Answer → 2 blocks south, 11 blocks west

Vector

quantity of both direction & magnitude, \vec{a}



magnitude

length of the vector,

$$|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

distance formula

direction

→ from initial point to terminal point

Component Form

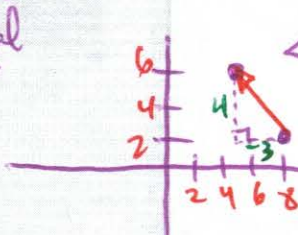
$\langle x, y \rangle$ horiz & vert change
 • weathermaps / animations

Matrix Form

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

EXAMPLE 1 Try It! Represent Vector Quantities

1. A vector has an initial point at (8, 2) and a terminal point at (5, 6). What is the vector in component form, and what are its magnitude and direction?



$$\langle x, y \rangle \rightarrow \langle 5 - 8, 6 - 2 \rangle$$

$$\vec{AB} \rightarrow \langle -3, 4 \rangle$$

$$|\vec{AB}| = \sqrt{(-3)^2 + 4^2} = 5$$

direction

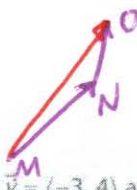
$\theta = \tan^{-1}(\frac{4}{-3})$
 make pos. $\theta = 53.1^\circ$ QII

→ 53.1° N of W
 → 36.9° W of N
 → 126.9°
 → -233.1°

EXAMPLE 2 Try It! Understand Vector Addition

2. a. If $\vec{MN} = \langle 9, 12 \rangle$ and $\vec{NO} = \langle 2, 7 \rangle$, what is $\vec{MN} + \vec{NO}$?

Tip to tail method/Rule

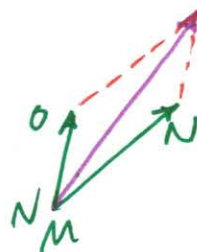


$$\vec{MN} + \vec{NO}$$

$$\rightarrow \langle 9 + 2, 12 + 7 \rangle$$

$$\rightarrow \langle 11, 19 \rangle$$

Parallelogram Method/Rule



b. If $\vec{v} = \langle -3, 4 \rangle$ and $\vec{w} = \langle 5, -8 \rangle$, what is $\vec{v} + \vec{w}$?



$$\vec{v} + \vec{w}$$

$$\rightarrow \langle -3 + 5, 4 + (-8) \rangle$$

$$\rightarrow \langle 2, -4 \rangle$$

EXAMPLE 3 Try It! Find the Magnitude and Direction of a Sum

3. If the engine speed was 9 mph northwest at 135° with the same current, what would be the magnitude and direction of the boat's speed? Round the magnitude and angle of direction to the nearest tenth.

HABITS OF MIND

Reason Let $\vec{v} = \langle -4, 12 \rangle$ and $\vec{w} = \langle -3, 9 \rangle$. Find $|\vec{v}| + |\vec{w}|$ and $|\vec{v} + \vec{w}|$. Does this mean that the sum of the magnitudes of two vectors is equal to the magnitude of their sum? Explain. © MP.2

No...

EXAMPLE 4 Try It! Understand Vector Subtraction

4. a. What are the components, magnitude, and direction of $\vec{s} - \vec{t}$, where $\vec{s} = \langle 6, -3 \rangle$ and $\vec{t} = \langle 3, 2 \rangle$?

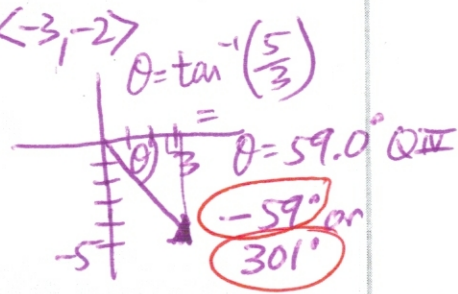
$$|\vec{s} - \vec{t}| = \sqrt{3^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.8$$

add opposites

$$\vec{s} - \vec{t} \rightarrow \vec{s} + (-\vec{t}) \rightarrow \langle 6, -3 \rangle + \langle -3, -2 \rangle = \langle 3, -5 \rangle$$

b. For $\vec{m} = \langle 1, -3 \rangle$ and $\vec{n} = \langle -2, 7 \rangle$, what is $\vec{m} - \vec{n}$?

$$\vec{m} - \vec{n} \rightarrow \langle 1, -3 \rangle + \langle 2, -7 \rangle = \langle 3, -10 \rangle$$

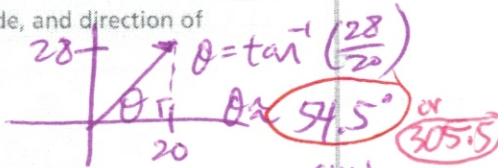


EXAMPLE 5 Try It! Multiply a Vector by a Scalar

5. a. If $\vec{t} = \langle -5, -7 \rangle$, what are the components, magnitude, and direction of $-4(\vec{t})$?

$$-4(\vec{t}) \rightarrow \langle -4(-5), -4(-7) \rangle = \langle 20, 28 \rangle$$

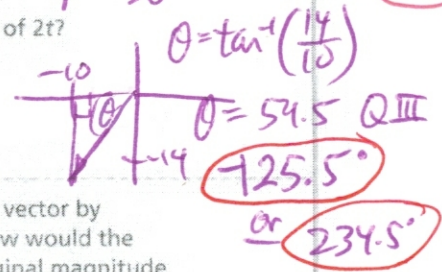
$$|-4(\vec{t})| = \sqrt{20^2 + 28^2} \approx 34.4$$



b. What are the components, magnitude, and direction of $2(\vec{t})$?

$$2(\vec{t}) \rightarrow \langle -10, -14 \rangle$$

$$|2(\vec{t})| = \sqrt{(-10)^2 + (-14)^2} \approx 17.2$$



HABITS OF MIND

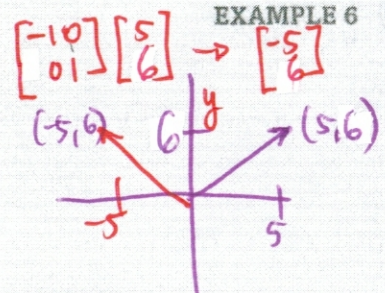
Make Sense and Persevere Suppose you were to multiply a vector by the scalar $\frac{1}{3}$. Subtract this result from the original vector. How would the magnitude and direction of the difference relate to the original magnitude and direction? © MP.1



$\frac{2}{3}$ rds of orig magnitude
but direction is the same

EXAMPLE 6 Try It! Use Matrices to Transform a Vector

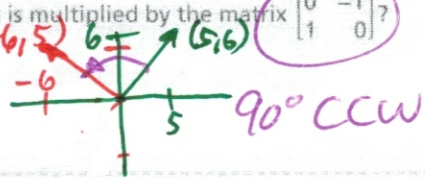
6. a. $\vec{EF} = \langle 5, 5 \rangle$. How is \vec{EF} transformed when it is multiplied by the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$?



change to <5, 6>
 $x \rightarrow -x$
 \rightarrow reflected horizontally / reflect about the y-axis

b. How is \vec{EF} transformed when it is multiplied by the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$?

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$$



HABITS OF MIND

Communicate Precisely How can a matrix and a vector be multiplied? © MP.6

vector written as a 2×1 matrix

$$\begin{bmatrix} - \\ - \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does including a direction with a quantity affect how you carry out operations on quantities?

2. **Error Analysis** Drew says the sum of the vectors $\vec{AB} = \langle 5, 11 \rangle$ and $\vec{BC} = \langle 2, -4 \rangle$ is $\vec{AC} = \langle 7, 13 \rangle$. Explain and correct Drew's error. © MP.3

3. **Communicate Precisely** Explain the process for vector subtraction. © MP.6

4. **Look for Relationships** Explain why you can use matrix multiplication to perform transformations on vectors. © MP.7

5. **Generalize** A boat is headed 60° north of west. In which quadrant is the boat? © MP.8

Do You KNOW HOW?

Write the component form of the vector, given its initial and terminal points.

6. initial point $(6, 2)$;
terminal point $(3, -5)$

$$\langle 3-6, -5-2 \rangle$$

$$\langle -3, -7 \rangle$$

7. initial point $(4, -1)$;
terminal point $(-8, 0)$

$$\langle -8-4, 0-(-1) \rangle$$

$$\langle -12, 1 \rangle$$

8. A vector has an initial point at $(6, 13)$ and a terminal point at $(3, 2)$. What is the vector in component form, and what are its magnitude and direction?

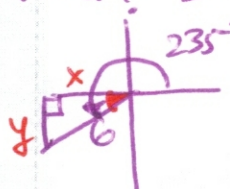
$$\langle 3-6, 2-13 \rangle \rightarrow \langle -3, -11 \rangle$$

$$|\langle -3, -11 \rangle| = \sqrt{(-3)^2 + (-11)^2} \approx 11.4$$

$\theta = \tan^{-1}\left(\frac{11}{3}\right) = 74.7^\circ$ QIII
 $+180^\circ$
 254.7° or 705.3°

9. A vector has a direction of 235° and a magnitude of 6. What is the component form of the vector? Express your answer to the nearest tenth of a unit.

TRIG? SOHCAHTOA



$$235^\circ \theta = 55^\circ \text{ QIII}$$

$$x = 6 \cos 55 = 3.4$$

$$y = 6 \sin 55 = 4.9$$

$$\langle -3.4, -4.9 \rangle$$

10. Find $\vec{MN} + \vec{NO}$ and $\vec{MN} - \vec{NO}$ if $\vec{MN} = \langle 6, 10 \rangle$ and $\vec{NO} = \langle -3, 0 \rangle$.

$$\langle 6 + (-3), 10 + 0 \rangle \rightarrow \langle 3, 10 \rangle$$

$$\langle 6 - (-3), 10 - 0 \rangle \rightarrow \langle 9, 10 \rangle$$