



10-4

Inverses and
Determinants**EXPLORE & REASON**

A teacher writes these three equations on the board.

- A. Carolina notices that the solution to the first equation is given by $\frac{3}{2}$, and she hypothesizes that

$$p + qi = \frac{1}{2 + 3i} \text{ and } \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Is Carolina correct?

$$\frac{2}{3} \cdot m = 1$$

$$(2 + 3i)(p + qi)$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- B. **Look for Relationships** What do the methods for solving these equations have in common? © MP.7

HABITS OF MIND

Communicate Precisely What does the term *multiplicative inverse* mean? © MP.6

**EXAMPLE 1** **Try It! Explore Inverses of 2×2 Matrices**

1. What is the inverse matrix of $\begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix}$?

EXAMPLE 2 **Try It! Find Inverses of Square Matrices**

2. Does each given matrix have an inverse? If so, find it.

a. $P = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix}$

b. $Q = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

c. $R = \begin{bmatrix} 5 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & 0 & 2 \end{bmatrix}$

EXAMPLE 3 **Try It! Use a Matrix Inverse**

3. The matrix $\begin{bmatrix} -3 & -5 & 11 & 6 \\ 130 & 105 & 106 & 65 \\ 323 & 267 & 205 & 128 \end{bmatrix}$ was encoded using the

matrix $A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$. What is the message?

HABITS OF MIND

Generalize What must be true in order for a matrix to have an inverse? © MP.8

EXAMPLE 4  **Try It!** Use Determinants to Find the Area of a Triangle

4. a. Find the area of the triangle determined by the vectors $\langle -2, 10 \rangle$ and $\langle -1, -5 \rangle$.
- b. Find the area of the triangle determined by the vectors $\langle 8, 4 \rangle$ and $\langle 7, -3 \rangle$.

EXAMPLE 5  **Try It!** Use a Determinant to Find the Area of a Parallelogram

5. Find the area of the parallelogram defined by the vectors $\langle 3, 8 \rangle$ and $\langle 1, 4 \rangle$.

HABITS OF MIND

Look for Relationships Find the area of the triangle defined by the vectors $\langle 2, 6 \rangle$ and $\langle -1, -3 \rangle$. How do you explain the result? © MP.7



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you find and use an inverse matrix?

2. **Vocabulary** What is the determinant of a 2×2 matrix?

3. **Error Analysis** Enrique says the matrix $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ has an inverse. Explain his error. © MP.3

4. **Communicate Precisely** Explain how to use the determinant of a matrix to find the area of a triangle. © MP.6

Do You KNOW HOW?

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} -2 & -4 \\ 2 & 3 \end{bmatrix}$

6. $\begin{bmatrix} -1 & 3 \\ -3 & 9 \end{bmatrix}$

7. $\begin{bmatrix} -3 & -2 & 1 \\ 5 & 4 & -3 \\ 6 & -4 & 2 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 & -4 \\ 0 & 6 & 3 \\ -1 & 1 & 3 \end{bmatrix}$

9. **Make Sense and Persevere** What is the area of a triangle determined by the vectors $\langle 2, 3 \rangle$ and $\langle 6, -1 \rangle$? © MP.1

10. What is the area of a parallelogram determined by the vectors $\langle 5, 2 \rangle$ and $\langle -1, -10 \rangle$?

