Activity

10-4 Inverses and Determinants

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EXPLORE & REASON

A teacher writes these three equations on the board.

$$\frac{2}{3} \cdot m = 1$$

$$(2+3i)(p+qi) = 1$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} w & x \\ y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A. Carolina notices that the solution to the first equation is given by $\frac{3}{2}$, and she hypothesizes that $p + qi = \frac{1}{2+3i}$ and $\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}}$.

Is Carolina correct?

$\cancel{\frac{3}{2}} \cdot \cancel{\frac{2}{3}} \cdot m = 1 \cdot \frac{3}{2}$

$\boxed{m = \frac{3}{2}}$

?

$\frac{(2+3i)(p+qi)}{(2+3i)} = \frac{1}{(2+3i)}$

$\boxed{(p+qi) = \frac{1}{2+3i}}$

Identity

$\boxed{\begin{bmatrix} w & x \\ y & z \end{bmatrix} = \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}{\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}}}$

Hmm....

B. **Look for Relationships** What do the methods for solving these equations have in common? MP.7

Each solution shows a multiplicative inverse....

HABITS OF MIND

Communicate Precisely What does the term *multiplicative inverse* mean? MP.6

ex) 16 & $\frac{1}{16}$ are multiplicative inverses..

because $16 \cdot \frac{1}{16} = 1$ Identity

reciprocal

A $\begin{bmatrix} a & b \end{bmatrix}$ • Square matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

• Square matrix

Notes

negatized

$\underset{2\times 2}{A^{-1}} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Inverse — determinant — swap — downhill − uphill

$\rightarrow \det A = ad - bc$

A^{-1} iff $\det A \neq 0$

$\underset{3\times 2}{A^{-1}} \rightarrow GC$

$\underset{3\times 3}{\det A} \rightarrow GC$: Run → OPTN → MAT → DET

Check

$A \cdot A^{-1} = I$

EXAMPLE 1

Try It! Explore Inverses of 2 × 2 Matrices

1. What is the inverse matrix of $\begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix}$?

A^{-1} exist?

$\det A = ad - bc$

$= 1(3) - 1(5) = -2 \leftarrow \neq 0 \ldots A^{-1}$ exists ...

$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} 3 & -5 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

$\begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} -\frac{3}{2} & \frac{5}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} + \frac{5}{2} & \frac{5}{2} - \frac{5}{2} \\ -\frac{3}{2} + \frac{3}{2} & \frac{5}{2} - \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ also for $A^{-1} \cdot A$

11 12 21 22

EXAMPLE 2

Try It! Find Inverses of Square Matrices

2. Does each given matrix have an inverse? If so, find it.

a. $P = \begin{bmatrix} -4 & 2 \\ -6 & 3 \end{bmatrix}$

P^{-1}? down up

$\det P = -4(3) - (-6)(2)$

$= -12 + 12 = 0$

P^{-1} DNE

b. $Q = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

Q^{-1}?

$\det Q = 7(1) - 2(3)$

$= 1$

Q^{-1} exists...

$Q^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$

c. $R = \begin{bmatrix} 5 & 1 & -1 \\ 2 & 0 & 5 \\ 1 & 0 & 2 \end{bmatrix}$

GC

$R^{-1} = \begin{bmatrix} 0 & -2 & 5 \\ 1 & 11 & -27 \\ 0 & 1 & -2 \end{bmatrix}$

?

matrix key — orig. matrix — Coded matrix

$A \times B = C$

$A^{-1} \times A \times B = A^{-1} \times C$

$I \times B = A^{-1} \times C$

$B = A^{-1} \times C$

EXAMPLE 3

Try It! Use a Matrix Inverse

Cryptography

Coded Matrix 3x4

3. The matrix $\begin{bmatrix} -3 & -5 & 11 & 6 \\ 130 & 105 & 106 & 65 \\ 323 & 269 & 205 & 128 \end{bmatrix}$ was encoded using the

Key/Encoder

matrix $A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix}$. What is the message?

3x3

$A^{-1} \leftarrow 3\times 3 \rightarrow GC$

$B = A^{-1} \times C$

$B = \begin{bmatrix} 12 & -7 & 3 \\ -20 & 12 & -5 \\ 1.5 & -1 & 0.5 \end{bmatrix}\begin{bmatrix} & & \\ & & \end{bmatrix}$

$\begin{bmatrix} 23 & 12 & 5 & 1 \\ 5 & 15 & 27 & 20 \\ 27 & 22 & 13 & 8 \end{bmatrix}$

A=1, B=2, C=3, ...

W L E A
E O ␣ T
␣ V M H

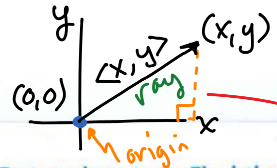
We love math

HABITS OF MIND

Generalize What must be true in order for a matrix to have an inverse? MP.8

• square matrix

• $\det \neq 0$



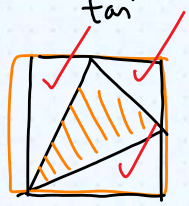
• direction
• magnitude

Notes

EXAMPLE 4 **Try It!** **Use Determinants to Find the Area of a Triangle**

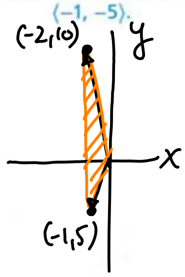
$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdots$ sin, cos, tan



subt other Δ's

4. a. Find the area of the triangle determined by the vectors ⟨−2, 10⟩ and ⟨−1, −5⟩.



area of Δ $= \frac{1}{2}\left|\det T\right|$ absolute value $T = \begin{bmatrix} -2 & -1 \\ 10 & -5 \end{bmatrix}$

$= \frac{1}{2}\left|(-2)(-5) - (10)(-1)\right|$

$= \frac{1}{2}\left|10 + 10\right| = \frac{1}{2}\left|20\right| = \boxed{10 \text{ units}^2}$

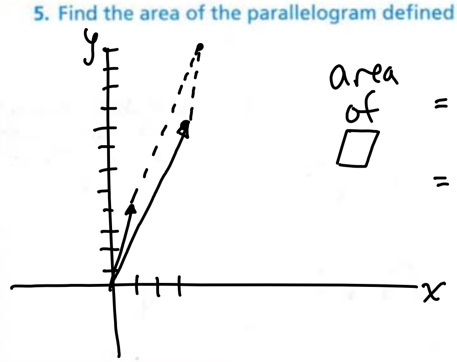
b. Find the area of the triangle determined by the vectors ⟨8, 4⟩ and ⟨7, −3⟩.

area of Δ $= \frac{1}{2}\left|\det T\right| = \frac{1}{2}\left|\det\begin{bmatrix} 8 & 7 \\ 4 & -3 \end{bmatrix}\right|$

$= \frac{1}{2}\left|8(-3) - 4(7)\right| = \frac{1}{2}\left|-24 - 28\right| = \frac{1}{2}\left|-52\right| = 26 \text{ units}^2$

EXAMPLE 5 **Try It!** **Use a Determinant to Find the Area of a Parallelogram**

5. Find the area of the parallelogram defined by the vectors ⟨3, 8⟩ and ⟨1, 4⟩.



area of ▱ $= \left|\det ▱\right|$

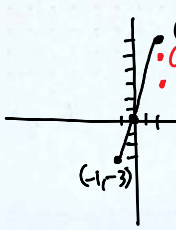
$= \left|\det\begin{bmatrix} 3 & 1 \\ 8 & 4 \end{bmatrix}\right|$

$\left|3(4) - 8(1)\right| = \left|12 - 8\right|$

$= \boxed{4 \text{ units}^2}$

HABITS OF MIND

Look for Relationships Find the area of the triangle defined by the vectors ⟨2, 6⟩ and ⟨−1, −3⟩. How do you explain the result? **MP.7**



• collinear
• Δ does not exist

area of Δ $= \frac{1}{2}\left|\det T\right| = \frac{1}{2}\left|\det\begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix}\right| = \frac{1}{2}\left|2(-3) - 6(-1)\right|$

$= \frac{1}{2}\left|-6 + 6\right| = \frac{1}{2}\left|0\right| = 0$???

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do you find and use an inverse matrix?

2. **Vocabulary** What is the determinant of a 2 × 2 matrix?

3. **Error Analysis** Enrique says the matrix $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ has an inverse. Explain his error. MP.3

4. **Communicate Precisely** Explain how to use the determinant of a matrix to find the area of a triangle. MP.6

Do You KNOW HOW?

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} -2 & -4 \\ 2 & 3 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-2(3)-2(-4)}\begin{bmatrix} 3 & 4 \\ -2 & -2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 4 \\ -2 & -2 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{3}{2} & 2 \\ -1 & -1 \end{bmatrix}}$$

6. $\begin{bmatrix} -1 & 3 \\ -3 & 9 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-1(9)-(-3)(3)}\begin{bmatrix} 9 & -3 \\ 3 & -1 \end{bmatrix}$$

$$= \frac{1}{-9+9} = 0 \quad A^{-1} \text{ DNE}$$

7. $\begin{bmatrix} -3 & -2 & 1 \\ 5 & 4 & -3 \\ 6 & -4 & 2 \end{bmatrix}$

GC : $\begin{bmatrix} -\frac{1}{6} & 0 & \frac{1}{12} \\ -\frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ -\frac{11}{6} & -1 & -\frac{1}{12} \end{bmatrix}$

8. $\begin{bmatrix} 2 & 0 & -4 \\ 0 & 6 & 3 \\ -1 & 1 & 3 \end{bmatrix}$

GC: $\begin{bmatrix} \frac{5}{2} & -\frac{2}{3} & 4 \\ -\frac{1}{2} & \frac{1}{3} & -1 \\ 1 & -\frac{1}{3} & 2 \end{bmatrix}$

9. **Make Sense and Persevere** What is the area of a triangle determined by the vectors $\langle 2, 3\rangle$ and $\langle 6, -1\rangle$? MP.1

$$\text{area of } \Delta = \frac{1}{2}\left|\det T\right| = \frac{1}{2}\left|\det\begin{bmatrix} 2 & 6 \\ 3 & -1 \end{bmatrix}\right|$$

$$= \frac{1}{2}\left|2(-1)-3(6)\right| = \frac{1}{2}\left|-2-18\right| = \frac{1}{2}\left|-20\right|$$

$$= \frac{1}{2}(20) = \boxed{10 \text{ units}^2}$$

10. What is the area of a parallelogram determined by the vectors $\langle 5, 2\rangle$ and $\langle -1, -10\rangle$?

$$\text{area of } \square = \left|\det \square\right| = \left|\det\begin{bmatrix} 5 & -1 \\ 2 & -10 \end{bmatrix}\right| = \left|5(-10)-2(-1)\right|$$

$$= \left|-50+2\right| = \left|-48\right| = \boxed{48 \text{ units}^2}$$