

CRITIQUE & EXPLAIN

This augmented matrix represents a system of linear equations in three variables. Students are asked to identify possible values of a and b for which the system has an infinite number of solutions.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 2 & 0 & 8 \\ 0 & 0 & a & b \end{array} \right]$$

Recall that an augmented matrix for a system of equations has a row for each equation showing its coefficients and constants.

Here are the answers that three students wrote:

Deshawn: $a = 0, b = 1$; Jacy: $a = 0, b = 0$; Avery: $a = 1, b = 0$

- A. Which student, if any, is correct? Explain your reasoning.
- B. For each student you think has an incorrect response, explain how many solutions their suggested values generate.
- C. **Look for Relationships** Which of the coefficient matrices that the three students wrote has an inverse? Is the number of solutions related to the existence of an inverse? © MP.7

HABITS OF MIND

Reason Why were you asked in part (c) to find the inverse of the coefficient matrices and not the augmented matrices? © MP.2

EXAMPLE 1  **Try It! Solve a Matrix Equation**

1. Solve the matrix equation $A \cdot X = B$ for $A = \begin{bmatrix} -1 & 4 & -2 \\ 2 & -1 & 0 \\ -1 & -4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$.

EXAMPLE 2  **Try It! Write a System of Linear Equations as a Matrix Equation**

2. Express each system of linear equations as a matrix equation.

a. $10x - 9y = 1$
 $7x + 6y = 13$

b. $4x + 2y - z = 14$
 $2x - 3y + 5z = 20$
 $3x - 6y = 8$

HABITS OF MIND

Make Sense and Persevere Justice found A^{-1} in the Try It for Example 1, and then multiplied BA^{-1} to solve the system. Why did Justice's calculator show an error message? © MP.1



EXAMPLE 3

**Try It!** Solve a System of Linear Equations Using an Inverse Matrix

3. Solve the following systems of linear equations using inverse matrices, if possible.

a.
$$\begin{cases} 3x + 4y = 8 \\ \frac{3}{2}x + 2y = 5 \end{cases}$$

b.
$$\begin{cases} x + 2y - 4z = 4 \\ x - 2y + 2z = -10 \\ -x - y + z = 4 \end{cases}$$

EXAMPLE 4

**Try It!** Solve a Real-World System With an Inverse

4. For a three-week period, the same company budgets \$860 for labor and \$1,080 for materials. How many pairs of men's and women's sneakers can they make in three weeks?

HABITS OF MIND

Generalize If the coefficient matrix for a system of equations does not have an inverse, does that mean that the system of equations has no solution? Explain. © MP.8

**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How can matrix inverses be used to simplify the process of solving a system of linear equations?

2. **Error Analysis** Corey says the matrix

$$\text{equation } \begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 22 \end{bmatrix}$$

represents the system of linear equations

$$\begin{cases} 3x + 2y = 8 \\ -y + 4z = 13 \\ 2x + 6z = 22 \end{cases}$$

Explain Corey's error. © MP3

3. **Vocabulary** How do you determine the coefficient matrix for a particular system of linear equations?

4. **Communicate Precisely** Explain how to solve a system of linear equations using an inverse matrix. © MP6

Do You KNOW HOW?

Express the system of linear equations as a matrix equation.

$$5. \begin{cases} 5x + 3y = -21 \\ 2x - 4y = -24 \end{cases}$$

$$6. \begin{cases} 6x - 8y + 2z = -46 \\ -x + 5y + 3z = 29 \\ 9x - 4z = -35 \end{cases}$$

7. Given the matrix equation $A \cdot X = B$ for

$$A = \begin{bmatrix} 1 & 3 & -4 \\ 2 & -2 & 3 \\ -4 & -6 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ -5 \\ -5 \end{bmatrix}, \text{ find } A^{-1}.$$

Then use A^{-1} to solve the matrix equation for X .

8. Write an equation that shows what your next step would be in solving this matrix equation

$$\text{for } x, y, \text{ and } z. \begin{bmatrix} -1 & 2 & -3 \\ 2 & -13 & 9 \\ -4 & 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$$

