Using Matrix Multiplication is faster than RREF

Activity

CRITIQUE & EXPLAIN

This augmented matrix represents a system of linear equations in three variables. Students are asked to identify possible values of a and b for which the system has an infinite number of solutions.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 2 & 0 & 8 \\ 0 & 0 & a & b \end{array}\right]$$

Recall that an augmented matrix for a system of equations has a row for each equation showing its coefficients and constants.

10-5 Inverse Matrices and Systems of Equations

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Here are the answers that three students wrote:

Deshawn: $a = 0$, $b = 1$; Jacy: $a = 0$, $b = 0$; Avery: $a = 1$, $b = 0$

A. Which student, if any, is correct? Explain your reasoning.

B. For each student you think has an incorrect response, explain how many solutions their suggested values generate.

C. **Look for Relationships** Which of the coefficient matrices that the three students wrote has an inverse? Is the number of solutions related to the existence of an inverse? MP.7

HABITS OF MIND

Reason Why were you asked in part (c) to find the inverse of the coefficient matrices and not the augmented matrices? MP.2

Notes

$AX = B$

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

Systems of Equations

graphing, substitution, elimination, RREF, Identity Matrix, operations

Assess

EXAMPLE 1 **Try It! Solve a Matrix Equation**

1. Solve the matrix equation $A \cdot X = B$ for $A = \begin{bmatrix} -1 & 4 & -2 \\ 2 & -1 & 0 \\ -1 & -4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$.

$AX = B$

$\begin{bmatrix} -1 & 4 & -2 \\ 2 & -1 & 0 \\ -1 & -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$

variable matrix

3x3 · 3x1 = 3x1

Coefficient matrix

Constant matrix

$A^{-1}AX = A^{-1}B$

I

$X = A^{-1}B$

$X = \begin{bmatrix} -\frac{1}{2} & 0 & -\frac{1}{2} \\ -1 & -1 & -1 \\ -2\frac{1}{4} & -2 & -1\frac{3}{4} \end{bmatrix} \begin{bmatrix} 6 \\ 8 \\ 2 \end{bmatrix}$

$X = \begin{bmatrix} -4 \\ -16 \\ -33 \end{bmatrix}$

$x = -4,\ y = -16,\ z = -33$

EXAMPLE 2 **Try It! Write a System of Linear Equations as a Matrix Equation**

2. Express each system of linear equations as a matrix equation.

a. $10x - 9y = 1$
$7x + 6y = 13$

$A \cdot X = B$

$\begin{bmatrix} 10 & -9 \\ 7 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \end{bmatrix}$

coef · var = const

b. $4x + 2y - z = 14$
$2x - 3y + 5z = 20$
$3x - 6y = 8$

$\begin{bmatrix} 4 & 2 & -1 \\ 2 & -3 & 5 \\ 3 & -6 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 8 \end{bmatrix}$

$X = A^{-1}B$

HABITS OF MIND

Make Sense and Persevere Justice found A^{-1} in the Try It for Example 1, and then multiplied BA^{-1} to solve the system. Why did Justice's calculator show an error message? MP.1

wrong dimensions

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Notes

Assess

$AX = B$

$A^{-1}AX = A^{-1}$

$X = A^{-1}B$

EXAMPLE 3 **Try It! Solve a System of Linear Equations Using an Inverse Matrix**

3. Solve the following systems of linear equations using inverse matrices, if possible.

a. $\begin{cases} 3x + 4y = 8 \\ \frac{3}{2}x + 2y = 5 \end{cases}$

$\begin{bmatrix} 3 & 4 \\ \frac{3}{2} & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$

A^{-1} DNE $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \text{error...} \end{bmatrix}\begin{bmatrix} 8 \\ 5 \end{bmatrix}$

- Check a point... (0,0): $0 + 0 = 8$, $0 + 0 = 5$ } False
  - no solution
  - parallel lines

b. $\begin{cases} x + 2y - 4z = 4 \\ x - 2y + 2z = -10 \\ -x - y + z = 4 \end{cases}$

$\begin{bmatrix} 1 & 2 & -4 \\ 1 & -2 & 2 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{2} & -\frac{1}{2} & -1 \\ -\frac{1}{2} & -\frac{1}{6} & -\frac{2}{3} \end{bmatrix}\begin{bmatrix} 4 \\ -10 \\ 4 \end{bmatrix}$

$X = \begin{bmatrix} -6 \\ -1 \\ -3 \end{bmatrix}$

$x = -6,\ y = -1,\ z = -3$

EXAMPLE 4 **Try It! Solve a Real-World System With an Inverse**

4. For a three-week period, the same company budgets $860 for labor and $1,080 for materials. How many pairs of men's and women's sneakers can they make in three weeks?

Women's Sneakers	Men's Sneakers
Labor $10	Labor $14
Materials $12	Materials $18

Let w = # women's
m = # men's

Labor Cost: $10w + 14m = 860$

Material Cost: $12w + 18m = 1080$

$\begin{bmatrix} 10 & 14 \\ 12 & 18 \end{bmatrix}\begin{bmatrix} w \\ m \end{bmatrix} = \begin{bmatrix} 860 \\ 1080 \end{bmatrix}$

$\begin{bmatrix} w \\ m \end{bmatrix} = \begin{bmatrix} 1.5 & -\frac{7}{6} \\ -1 & \frac{5}{6} \end{bmatrix}\begin{bmatrix} 860 \\ 1080 \end{bmatrix}$

$\begin{bmatrix} w \\ m \end{bmatrix} = \begin{bmatrix} 30 \\ 40 \end{bmatrix}$:)

30 pairs of women's sneakers...
40 " " men's

HABITS OF MIND

Generalize If the coefficient matrix for a system of equations does not have an inverse, does that mean that the system of equations has no solution? Explain. MP.8

or

skew lines

Assess

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can matrix inverses be used to simplify the process of solving a system of linear equations?

2. **Error Analysis** Corey says the matrix equation $\begin{bmatrix} 3 & 2 \\ -1 & 4 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 22 \end{bmatrix}$ represents the system of linear equations $\begin{cases} 3x + 2y = 8 \\ -y + 4z = 13. \\ 2x + 6z = 22 \end{cases}$ Explain Corey's error. MP.3

3. **Vocabulary** How do you determine the coefficient matrix for a particular system of linear equations?

4. **Communicate Precisely** Explain how to solve a system of linear equations using an inverse matrix. MP.6

Do You KNOW HOW?

Express the system of linear equations as a matrix equation.

5. $\begin{cases} 5x + 3y = -21 \\ 2x - 4y = -24 \end{cases}$

$\begin{bmatrix} 5 & 3 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -21 \\ -24 \end{bmatrix}$

6. $\begin{cases} 6x - 8y + 2z = -46 \\ -x + 5y + 3z = 29 \\ 9x - 4z = -35 \end{cases}$

$\begin{bmatrix} 6 & -8 & 2 \\ -1 & 5 & 3 \\ 9 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -46 \\ 29 \\ -35 \end{bmatrix}$

7. Given the matrix equation $A \cdot X = B$ for GC 3x3 $A = \begin{bmatrix} 1 & 3 & -4 \\ 2 & -2 & 3 \\ -4 & -6 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ -5 \\ -5 \end{bmatrix}$, find A^{-1}. Then use A^{-1} to solve the matrix equation for X.

$A^{-1} = \begin{bmatrix} -\frac{2}{7} & \frac{27}{70} & \frac{1}{70} \\ -\frac{1}{7} & -\frac{17}{70} & -\frac{11}{70} \\ -\frac{2}{7} & -\frac{3}{35} & -\frac{4}{35} \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} 0 \\ -5 \\ -5 \end{bmatrix}$

$X = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

$x = -2, y = 2, z = 1$

8. Write an equation that shows what your next step would be in solving this matrix equation for x, y, and z. $\begin{bmatrix} -1 & 2 & -3 \\ 2 & -13 & 9 \\ -4 & 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$

A^{-1}?

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} & -\frac{4}{11} & -\frac{7}{22} \\ -\frac{4}{11} & \frac{7}{11} & \frac{1}{22} \\ -\frac{14}{33} & \frac{2}{33} & \frac{3}{22} \end{bmatrix} \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$x = 1, y = 0, z = -1$

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