

2-3
Factored Form of a Quadratic Function
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CRITIQUE & EXPLAIN Intercept

Corey wrote an equation in **factored form** $y = (x + 8)(x - 2)$, to represent a quadratic function. Kimberly wrote the equation $y = x^2 + 6x - 16$ and Joshua wrote the equation $y = (x + 3)^2 - 25$.

A. Reason Do all three equations represent the same function? If not, whose is different? Explain algebraically.

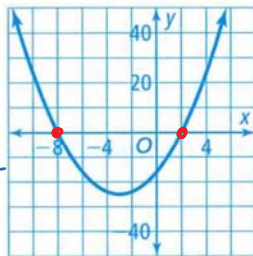
• All the same...

Vertex

$$(x+3)(x+3) - 25$$

$$x^2 + 3x + 3x + 9 - 25$$

$$x^2 + 6x - 16$$



Standard

$$(x+8)(x-2)$$

$$x^2 - 2x + 8x - 16$$

$$x^2 + 6x - 16$$

B. How else could you determine if all three equations represent the same function?

- GC
- Subst in values ...
- vertex, y-int, x-int ...

C. What information can Corey's form help you find that is more difficult to find using Kimberly's or Joshua's form?

- X-intercepts
- aka zeros, roots, solutions

HABITS OF MIND

Use Structure Whose form of the equation is most useful for finding the vertex? The y-intercept? The x-intercepts? © MP.7

KIMBERLY COREY

Josh

Factor $ax^2+bx+c \rightarrow (x+m)(x+n)$ or $y=a(x-p)(x-q)$

ex) Factor x^2-x-12 $a=1, b=-1, c=-12$

• Guess'n'check
FOIL
 $b: -1$

Notes
 $(x-1)(x+12) \rightarrow 12x-1x$
 $(x-2)(x+6) \rightarrow 6x-2x$
 $(x-3)(x+4) \rightarrow 4x-3x$
 $(x-4)(x+3) \rightarrow 3x-4x$

$x^2+xn+xm+mn$
 $x^2+(m+n)x+mn$
 $a+c$ ac

Try It! Factor a Quadratic Expression

1. Factor the expression.

a. x^2-9 • diff of squares
 $\rightarrow (x+3)(x-3)$
 or x^2+0x-9
 diamond method
 -3 3
 0 -9
 ac product

b. $3x^2-7x+2$
 a b c
 $3x^2$ $-7x$ $+2$
 $3x$ -1
 $-2x$ 2
 $-7x$
 $6x^2$
 $-1x, -6x$
 $-2x, -3x$
 $\rightarrow (3x-1)(x-2)$

OR -7
 -6 -1
 6

OR Factor by Grouping
 $3x^2-1x-6x+2$
 $x(3x-1)-2(3x-1)$
 $(3x-1)(x-2)$

factors of ac method

$(1)(-12)$
 $ac = -12$ $b = -1$

-1, 12	-1+12=11
-2, 6	-2+6=4
-3, 4	-3+4=1
-4, 3	-4+3=-1

Sum of those factors \rightarrow otherwise factor neg out
 $x^2-4x+3x-12$
 • factor by grouping

• ac box method
 \rightarrow GCF rows & column

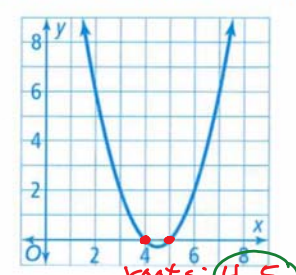
x^2-4x
 $+3x-12$
 x -4
 3 -12

• If leading term is neg, then factor it out too

EXAMPLE 2 Try It! Relate Factors to Zeros of a Function

2. The graph shows the function $y=x^2-9x+20$. Identify the zeros of the function. How do the zeros relate to the factors of $x^2-9x+20$?

$x^2-9x+20$
 x -9
 5 20
 $-9x$
 $20x^2$
 $-1x, -20x$
 $-2x, -10x$
 $-4x, -5x$



x ints/roots/sols

$(x-5)(x-4)$
 $a(x-p)(x-q)$
 roots

Hybrid x^2-x-12

x^2-x-12
 x -4
 3 -12
 $-x$
 $-12x$
 $-1x, 12x$
 $-2x, 6x$
 $-3x, 4x$
 $-4x, 3x$

- 1st term: 1st box Last term: last box Middle term: bottom left corner...
- Product of downhill...
- Find the sum of the factors of product...

- place factors into box
- GCF row & column

EXAMPLE 3 Try It! Solve Quadratic Equations by Factoring

3. Solve the equation by factoring.

a. $x^2+8x=20$
 $x^2+8x-20=0$
 $(x-2)(x+10)=0$
 $x-2=0$ or $x+10=0$
 $x=2$ or $x=-10$

b. $2x^2-3x-2=0$
 $2x^2$ $-3x$ -2
 $2x$ $+1$
 -2 -2
 $-3x$
 $-4x^2: -1x, 4x$
 $1x, -4x$
 $-2x, 2x$

$(2x+1)(x-2)=0$
 $2x+1=0$ $x-2=0$
 $x=-\frac{1}{2}$ $x=2$
 $-2x^2+3x+2=0$
 $-1(2x^2-3x-2)=0$

HABITS OF MIND

Error Analysis Anna solved the equation $x^2+8x-20=0$ by factoring. She wrote $(x-10)(x+2)=0$ and expected the x-intercepts of the function $y=x^2+8x-20$ to be at 10 and 2 . Was she right? © MP3

$(x-p)(x-q)$ No... p & q are opposite
 10 -2

$(x-p)(x-q)$ No... p & q are opposite
 10 -2

Notes

Assess

EXAMPLE 4 Try It! Find the Zeros of a Quadratic Function

4. A baseball is thrown from the upper deck of a stadium, 108 ft above the ground. The function $h(t) = -16t^2 + 32t + 128$ gives the height of the ball t seconds after it is thrown. How long will it take the ball to reach the ground?

$h(t) = 0$ height: 0

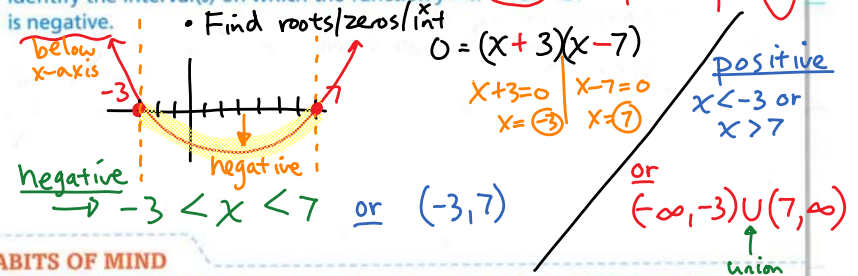
$$-16(t^2 - 2t - 8)$$

$$-16(t+2)(t-4) = 0$$

$t = -2$ | $t = 4$
 4 second

EXAMPLE 5 Try It! Determine Positive or Negative Intervals

5. Identify the interval(s) on which the function $y = x^2 - 4x - 21$ is negative.



HABITS OF MIND

Construct Arguments Is it always true that the y-values of a quadratic function have opposite signs on either side of a zero of the function? Explain why or give a counterexample. © MP.3



EXAMPLE 6 Try It! Write the Equation of a Parabola in Factored Form

6. Write an equation of a parabola with x-intercepts at (3, 0) and (-3, 0) and which passes through the point (1, 2).

stretch/shrink/refl

Intercept

$p: 3$ $q: -3$

$$y = a(x-p)(x-q)$$

$$2 = a(1-3)(1-(-3))$$

$$2 = a(-2)(4)$$

$$2 = -8a$$

$$\frac{-1}{4} = a$$

$$y = -\frac{1}{4}(x-3)(x+3)$$

HABITS OF MIND

Model With Mathematics Is there any other parabola with x-intercepts at (-2, 0) and (-1, 0)? Give an equation or explain why there is no such

HABITS OF MIND

Model With Mathematics Is there any other parabola with x-intercepts at $(-2, 0)$ and $(-1, 0)$? Give an equation or explain why there is no such parabola. © MP.4

a value : stretchiness....

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How is the factored form helpful in solving quadratic equations?

2. **Error Analysis** Amir says the graph of $y = x^2 + 16$ has -4 as a zero. Is Amir correct? Explain. **MP.3**

3. **Vocabulary** How does the factored form of a quadratic equation relate to the Zero Product Property? **MP.7**

4. **Generalize** How does knowing the zeros of a function help determine where a function is positive? **MP.8**

Do You KNOW HOW?

Factor each expression.

5. $x^2 - 5x - 24$

6. $5x^2 + 3x - 2$

The work shows a cross-multiplication method with numbers 3, -2, -10, and 5. A 2x2 grid contains $5x^2$, $5x$, $-2x$, and -2 . The final factored form is $(5x-2)(x+1)$ and the expanded form $5x^2 + 5x - 2x - 2$ is written below.

Solve each equation.

7. $x^2 = 12x - 20$

8. $4x^2 - 5x - 6 = 0$

The work shows a cross-multiplication method with numbers -5, -6, -24, and 3. A 2x2 grid contains $4x^2$, $-8x$, $3x$, and -6 . The factored form is $(4x+3)(x-2) = 0$, leading to solutions $x = -\frac{3}{4}$ and $x = 2$.

9. The height, in feet, of a t-shirt launched from a t-shirt cannon high in the stands at a football stadium is given by $h(x) = -16x^2 + 64x + 80$, where x is the time in seconds after the t-shirt is launched. How long will it take before the t-shirt reaches the ground?