Activity

2-4 Complex Numbers and Operations

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EXPLORE & REASON

A math class played a game called "Solve It, You're Out." At the start of each round, students chose a card from a deck marked with integers from −5 to 5. When an equation is shown, any student whose card states the solution to the equation is eliminated. Five students remain.

Five remaining students with the cards they hold:
Mercedes: −3
Steve: 0
Aubrey: 1
Elijah: −2
Fatima: 3

A. The next equation presented was $x^2 = 9$. Which student(s) was eliminated? Explain.

B. **Construct Arguments** In the next round, the equation presented was $x^2 = -4$. Darius thought he was eliminated, but this is not the case. Explain why Darius was incorrect. MP.3

C. What is true about solutions to $x^2 = a$ when a is a positive number? When a is a negative number? What about when $a = 0$?

HABITS OF MIND

Reason Steve thought that he was a sure winner because he could not be eliminated. Is he correct? Explain. If not, write an equation of the form $x^2 = a$ that would eliminate Steve. MP.2

Notes

Assess

EXAMPLE 1 **Try It! Solve a Quadratic Equation Using Square Roots**

1. Use square roots to solve each equation. Write your solutions using the imaginary unit, i.

   a. $x^2 = -5$

   b. $x^2 = -72$

HABITS OF MIND

Communicate Precisely How do you know that the solution to the equation $x^2 = -5$ must be an imaginary number? MP.6

EXAMPLE 2 **Try It! Add and Subtract Complex Numbers**

2. Find the sum or difference.

   a. $(-4 + 6i) + (-2 - 9i)$

   b. $(3 - 2i) - (-4 + i)$

HABITS OF MIND

Generalize How is adding and subtracting complex numbers similar to adding and subtracting binomials? MP.8

EXAMPLE 3 **Try It! Multiply Complex Numbers**

3. Write each product in the form $a + bi$.

   a. $\frac{2}{5}i\left(10 - \frac{5}{2}i\right)$

   b. $\left(\frac{1}{2} + 2i\right)\left(\frac{1}{2} - 2i\right)$

EXAMPLE 4 **Try It! Simplify a Quotient With Complex Numbers**

4. Write each quotient in the form $a + bi$.

   a. $\frac{80}{2 - 6i}$

   b. $\frac{4 - 3i}{-1 + 2i}$

HABITS OF MIND

Use Structure Why do you multiply the numerator and denominator of a complex fraction by the conjugate of the denominator? MP.7

EXAMPLE 5 **Try It! Factor a Sum of Squares**

5. Factor each expression.

   a. $4x^2 + 25$

   b. $8y^2 + 18$

EXAMPLE 6 **Try It! Solve a Quadratic Equation With Complex Solutions**

6. Find the value(s) of x that will solve each equation.

   a. $x^2 + 49 = 0$

   b. $9x^2 + 25 = 0$

HABITS OF MIND

Construct Arguments For what values of a will the solutions to the equation $x^2 - a = 0$ be complex numbers? Explain how you know. MP.3

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you represent and operate on numbers that are not on the real number line?

2. **Vocabulary** How do you form the *complex conjugate* of a complex number $a + bi$?

3. **Error Analysis** Zora was asked to write the quotient $\frac{4}{3-i}$ in the form $a + bi$. She began this way: $\frac{4}{3-i} \times \frac{3-i}{3-i} = \frac{4(3-i)}{3^2+1^2} = \frac{12-4i}{10}$. Explain the error Zora made. **MP.3**

4. **Look for Relationships** The quadratic equation $x^2 + 9 = 0$ has solutions $x = 3i$ and $x = -3i$. How many times will the graph of $f(x) = x^2 + 9$ cross the x-axis? Explain. **MP.7**

Do You KNOW HOW?

Write each of the following in the form $a + bi$.

5. $(2 + 5i) - (-6 + i)$

6. $(2i)(6 + 3i)$

Solve each equation.

7. $x^2 + 16 = 0$

8. $y^2 = -25$

9. The total source voltage in the circuit is $6 - 3i$ volts. What is the voltage at the middle source? **MP.4**