Activity

2-4 Complex Numbers and Operations

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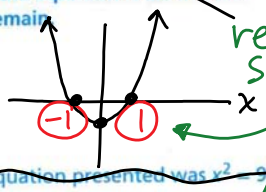
EXPLORE & REASON

A math class played a game called "Solve It, You're Out." At the start of each round, students chose a card from a deck marked with integers from −5 to 5. When an equation is shown, any student whose card states the solution to the equation is eliminated. Five students remain.

Five remaining students with the cards they hold:
Mercedes: −3
Steve: 0
Aubrey: 1
Elijah: −2
Fatima: 3

Graph : x^2-1

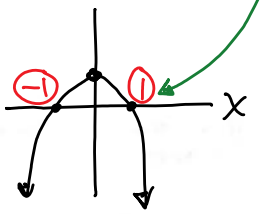
zeros/roots/x-ints



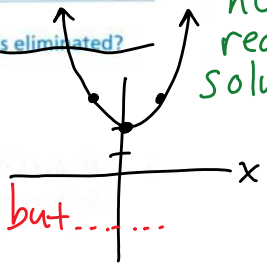
real solutions

A. The next equation presented was $x^2 = 9$. Which student(s) was eliminated? Explain.

$-x^2+1$



x^2+2



no real solutions

but.....

B. Construct Arguments In the next round, the equation presented was $x^2 = -4$. Darius thought he was eliminated, but this is not the case. Explain why Darius was incorrect. MP.3

$\sqrt{\text{negative \#}}$ → error : real number system

However... Let's define $\sqrt{-1} = i$ ← imaginary unit

- treat i like a variable

C. What is true about solutions to $x^2 = a$ when a is a positive number? When a is a negative number? What about when $a = 0$?

→ Complex Number

$a+bi$

real Imaginary

HABITS OF MIND

Reason Steve thought that he was a sure winner because he could not be eliminated. Is he correct? Explain. If not, write an equation of the form $x^2 = a$ that would eliminate Steve. MP.2

$i = \sqrt{-1}$ imaginary unit $a+bi$ (real, imag)

Notes

ex 1a) Solve $x^2 = 16$

- $x = 4$ or -4
- $x = \pm 4$

$\sqrt{x^2} = \pm\sqrt{16}$

$x = \pm 4$

- graph
- quad formula...

ex 1b) Solve $x^2 = -9$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm\sqrt{-1 \cdot 9}$

$x = \pm\sqrt{-1}\sqrt{9}$

$x = \pm 3i$

EXAMPLE 1 $x^2 - 16 = 0$ (diff of squares)

$(x-4)(x+4) = 0$

$x = 4$ | $x = -4$

Try It! Solve a Quadratic Equation Using Square Roots

1. Use square roots to solve each equation. Write your solutions using the imaginary unit, i.

a. $x^2 = -5$

$\sqrt{x^2} = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

or $\pm\sqrt{5}i$

b. $x^2 = -72$

$\sqrt{x^2} = \pm\sqrt{-72}$

$x = \pm\sqrt{-1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

$x = \pm 6i\sqrt{2}$

- simplified

HABITS OF MIND

Communicate Precisely How do you know that the solution to the equation $x^2 = -5$ must be an imaginary number? MP.6

$\sqrt{\text{neg}}$ = imaginary #

- combine like parts of complex # $a+bi$ (real, imag)

EXAMPLE 2

Try It! Add and Subtract Complex Numbers $a+bi$

2. Find the sum or difference.

a. $(-4 + 6i) + (-2 - 9i)$

$-6 - 3i$

b. $(3 - 2i) - (-4 + i)$

$3 - 2i + 4 - i$

$7 - 3i$

HABITS OF MIND

Generalize How is adding and subtracting complex numbers similar to adding and subtracting binomials? MP.8

- Combine like terms/parts → real & imag

Notes

$i = \sqrt{-1}$

$i^2 = \sqrt{-1}\cdot\sqrt{-1} = -1$

$i^3 = i\cdot i^2 = i\cdot(-1) = -i$

$i^4 = i^2\cdot i^2 = (-1)(-1) = 1$

$i^5 = i^4\cdot i = i^1$

→ cycles of 4

ex) $i^{253} = 4\overline{)253}\ R\ 1 \rightarrow i^1$ (63; 24; 13)

Complex Conjugate

$a+bi$ & $a-bi$

→ FOIL

$i^2 = -1$

Assess

EXAMPLE 3 **Try It! Multiply Complex Numbers**

3. Write each product in the form $a + bi$.

a. $\frac{2}{5}i\left(10 - \frac{5}{2}i\right)$ distrib

$\rightarrow \frac{2}{5}i(10) - \frac{2}{5}i\left(\frac{5}{2}i\right)$

$\rightarrow 4i - i^2 \rightarrow 4i - (-1) \rightarrow 4i + 1$

or $1 + 4i$

b. $\left(\frac{1}{2} + 2i\right)\left(\frac{1}{2} - 2i\right)$ FOIL

$\rightarrow \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}(-2i) + 2i\left(\frac{1}{2}\right) + 2i(-2i)$

$\rightarrow \frac{1}{4} - i + i - 4i^2$ ($i^2 = -1$)

$\rightarrow \frac{1}{4} - 4(-1) \rightarrow \frac{1}{4} + 4 \rightarrow \frac{17}{4}$

EXAMPLE 4 **Try It! Simplify a Quotient With Complex Numbers**

4. Write each quotient in the form $a + bi$.

a. $\frac{80}{2 - 6i}$ multiply by complex conjugate

$\rightarrow \frac{80}{2(1-3i)} \rightarrow \frac{40}{1-3i} \cdot \frac{(1+3i)}{(1+3i)} \rightarrow \frac{40 + 120i}{1 + 3i - 3i - 9i^2}$ ($i^2 = -1$)

$\rightarrow \frac{40 + 120i}{1 + 9} \rightarrow \frac{40 + 120i}{10} \rightarrow \frac{40}{10} + \frac{120}{10}i \rightarrow 4 + 12i$

b. $\frac{4 - 3i}{-1 + 2i}$ FOIL $\cdot \frac{(-1-2i)}{(-1-2i)} \rightarrow \frac{-4 - 8i + 3i + 6i^2}{1 \ldots -4i^2}$ ($i^2 = -1$)

$\rightarrow \frac{-4 - 5i - 6}{1 + 4} \rightarrow \frac{-10 - 5i}{5} \rightarrow -\frac{10}{5} - \frac{5}{5}i \rightarrow -2 - 1i$

HABITS OF MIND

Use Structure Why do you multiply the numerator and denominator of a complex fraction by the conjugate of the denominator? MP.7

→ eliminate Imag #s in denominator

…remember difference of squares?

Factor $x^2 + 9$

- binomial
- diff
- squarootable

$(x+3)(x-3)$

Conjugates $ax+b$ & $ax-b$

EXAMPLE 5 **Try It! Factor a Sum of Squares**

5. Factor each expression.

a. $4x^2 + 25$ diff?

$\rightarrow (4x^2 - -25)$

$\rightarrow (2x + 5i)(2x - 5i)$

b. $8y^2 + 18$ • mono GCF?

$\rightarrow 2(4y^2 - -9)$

$\rightarrow 2(2y + 3i)(2y - 3i)$

EXAMPLE 6 **Try It! Solve a Quadratic Equation With Complex Solutions**

6. Find the value(s) of x that will solve each equation.

a. $x^2 + 49 = 0$ Square root… • Factor diff of squares

$x^2 + 49 = 0$ | $x^2 - -49 = 0$

$-49 \quad -49$ | $(x + 7i)(x - 7i) = 0$

$\sqrt{x^2} = \sqrt{-49}$ | $x + 7i = 0 \quad x - 7i = 0$

$x = \pm 7i$ | $x = -7i \quad x = 7i$

b. $9x^2 + 25 = 0$

$9x^2 = -25$

$x^2 = -\frac{25}{9}$

$x = \pm\sqrt{\frac{-25}{9}}$

$x = \pm\frac{5i}{3}$

$9x^2 - -25 = 0$

$(3x + 5i)(3x - 5i) = 0$

$3x + 5i = 0 \quad 3x - 5i = 0$

$x = \frac{-5i}{3} \quad x = \frac{5i}{3}$

HABITS OF MIND

Construct Arguments For what values of a will the solutions to the equation $x^2 - a = 0$ be complex numbers? Explain how you know. MP.3

Assess

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you represent and operate on numbers that are not on the real number line?

2. **Vocabulary** How do you form the *complex conjugate* of a complex number $a + bi$?

3. **Error Analysis** Zora was asked to write the quotient $\frac{4}{3-i}$ in the form $a + bi$. She began this way: $\frac{4}{3-i} \times \frac{3-i}{3-i} = \frac{4(3-i)}{3^2+1^2} = \frac{12-4i}{10}$. Explain the error Zora made. MP.3

4. **Look for Relationships** The quadratic equation $x^2 + 9 = 0$ has solutions $x = 3i$ and $x = -3i$. How many times will the graph of $f(x) = x^2 + 9$ cross the x-axis? Explain. MP.7

Do You KNOW HOW?

Write each of the following in the form $a + bi$.

5. $(2 + 5i) - (-6 + i)$

→ $2 + 5i + 6 - i$

→ $8 + 4i$

6. $(2i)(6 + 3i)$

→ $12i + 6i^2$ ($i^2 = -1$)

→ $12i - 6$ or $-6 + 12i$

Solve each equation.

7. $x^2 + 16 = 0$

Square root

$x^2 = -16$

$x = \pm\sqrt{-16}$

$x = \pm 4i$

diff of squares

$x^2 - 16 = 0$

$(x + 4i)(x - 4i) = 0$

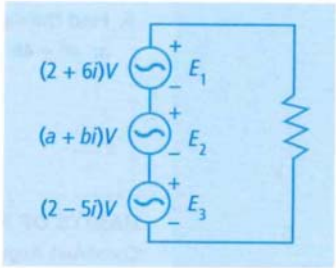
$x = -4i$ | $x = 4i$

8. $y^2 = -25$

$y = \pm\sqrt{-25}$

$y = \pm 5i$

9. The total source voltage in the circuit is $6 - 3i$ volts. What is the voltage at the middle source? MP.4



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