

Standard  $ax^2+bx+c$   $\rightarrow$  Vertex  $a(x-h)^2+k$



**2-5**  
Completing the Square  
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**CRITIQUE & EXPLAIN**

Hana and Enrique used different methods to solve the equation  $x^2 - 6x + 9 = 16$ .

Factor & Solve...

**Hana**  
 $x^2 - 6x + 9 = 16$   
 $x^2 - 6x - 7 = 0$   
 $(x-7)(x+1) = 0$   
 $x-7=0$  OR  $x+1=0$   
 $x=7$  OR  $x=-1$   
 The solutions are 7 and -1.

**Enrique**  
 PST  $\rightarrow$  Perfect Square Trinomial  
 $x^2 - 6x + 9 = 16$   
 $(x-3)(x-3) = 16$   
 $(x-3)^2 = 16$   
 I can square 4 or -4 to get 16.  
 $x-3=4$  OR  $x-3=-4$   
 $x=7$  OR  $x=-1$   
 The solutions are 7 and -1.

Both methods work

A. Does Hana's method work? If her method is valid, explain the reasoning she used. If her method is not valid, explain why not. © MP2

• Factor & Solve

B. Does Enrique's method work? If his method is valid, explain the reasoning he used. If his method is not valid, explain why not.

• Factored the Perfect Square Trinomial & Solved by Square Root ...

C. Use Structure Can you use either Hana's or Enrique's method to solve the equation  $x^2 + 10x + 25 = 3$ ? Explain. © MP7

PST because  $\sqrt{25} = 5$   
 double 5 = 10

**HABITS OF MIND**

**Make Sense and Persevere** Why does Hana set her two factors equal to zero, while Enrique sets his factor equal to 4 and -4? © MP1

$y=4$  or  $-4$

$x$ -axis  
 $y=0$

Perfect Square Trinomial

Perfect Square Trinomial

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Notes

Assess

EXAMPLE 1 Try It! Use Square Roots to Solve Quadratic Equations

1. Find the solution(s) to the equations.

a.  $81 = x^2 + 12x + 36$  ? double b.  $9 = x^2 - 16x + 64$

Factor PST

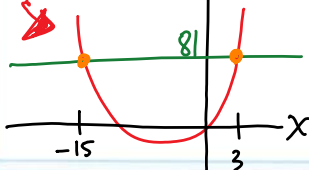
81 =  $(x+6)^2$   
 $\pm\sqrt{81} = \sqrt{(x+6)^2}$   
 $\pm 9 = x+6$   
 $\pm 9 - 6 = x + 6 - 6$   
 $3, -15 = x$

$9 = (x-8)(x-8)$   
 $9 = (x-8)^2$   
 $\pm\sqrt{9} = \sqrt{(x-8)^2}$   
 $\pm 3 = x-8$   
 $8 \pm 3 = x$   
 $11, 5 = x$

HABITS OF MIND

Use Structure How do you recognize a perfect square trinomial? © MP7

See above



Complete the Square Process

$$ax^2 + bx + c \rightarrow a(x-h)^2 + k$$

$a=1$ ... If  $a \neq 1$ , then factor out from  $ax^2 + bx$

$(\frac{b}{2})^2$  "Poof"  $\rightarrow$  PST

EXAMPLE 2 Try It! Understand the Process of Completing the Square

2. How can you write the equation  $x^2 - 6x - 11 = 0$  in the form  $(x-p)^2 = q$ ?

$x^2 - 6x + 9 = 11 + 9$  balance out eqn.  $+11 +11$   
 $(x-3)(x-3)$   
 $(x-3)^2 = 20$

EXAMPLE 3 Try It! Solve a Quadratic Equation by Completing the Square (Inefficient)

3. Solve the following equations by completing the square.

a.  $0 = x^2 + 4x + 8$

b.  $0 = x^2 - 8x + 17$

$+4 - 8 = x^2 + 4x + 4$   
 $-4 = (x+2)(x+2)$   
 $-4 = (x+2)^2$   
 $\pm\sqrt{-4} = \sqrt{(x+2)^2}$   
 $-2 \pm 2i = x$

$-17 + 16 = x^2 - 8x + 16$   
 $-1 = (x-4)^2$   
 $\pm\sqrt{-1} = \sqrt{(x-4)^2}$   
 $\pm i = x-4$   
 $4 \pm i = x$

HABITS OF MIND

Reason Richard is completing the square to solve the equation  $2x^2 + 8x = 19$ . He wrote:  $2(x^2 + 4x + 4) = 19 + \underline{\hspace{2cm}}$ . What number should Richard write in the blank? © MP2

$2(x^2 + 4x + 4) = 19 + 8$

Notes

Assess

**EXAMPLE 4**  **Try It!** Complete the Square to Solve a Real-World Problem



4. The relationship between the time since the ball was thrown and height of the ball can be modeled by the equation  $h = 32t - 16t^2$ , where  $h$  is the height of the ball after  $t$  seconds. Complete the square to find how long it will take the ball to reach a height of 20 ft.

**EXAMPLE 5**  **Try It!** Write a Quadratic Equation in Vertex Form

5. Write each equation in vertex form. Identify the maximum or minimum value of the graph of each equation.

a.  $y = -3x^2 - 9x + 7$

$a \neq 1$

$$y = -3\left(x^2 + 3x + \frac{9}{4}\right) + 7$$

$$y = -3\left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right) + 7 + 3\left(\frac{9}{4}\right)$$

$$= -3\left(x + \frac{3}{2}\right)^2 + \frac{55}{4}$$

$V: \left(-\frac{3}{2}, \frac{55}{4}\right)$  maximum

b.  $y = 2x^2 + 12x + 9$

$a \neq 1$

$$y = 2(x^2 + 6x + 9) + 9 - 2 \cdot 9$$

PST

$$y = 2(x + 3)(x + 3) + 9 - 18$$

$$y = 2(x + 3)^2 - 9$$

$V: (-3, -9)$  minimum

**HABITS OF MIND**

**Make Sense and Persevere** A pelican swoops down under the surface of the ocean to catch a fish. An equation that describes the pelican's path is  $y = 4x^2 - 16x + 15$ . The pelican catches the fish at the deepest point of the dive. How deep was the fish swimming? © MP.1

### Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve a quadratic equation by completing the square?

2. Paula said that only quadratic equations with leading coefficients of 1 can be solved by completing the square. Is Paula correct? Explain. **MP.3**

3. **Generalize** Given the expression  $x^2 + bx$ , describe how to find  $c$  so that  $x^2 + bx + c$  is a perfect square trinomial. **MP.8**

4. **Make Sense and Persevere** How can you complete the square to find the vertex of a parabola? **MP.1**

### Do You KNOW HOW?

Solve each equation by completing the square.

5.  $0 = x^2 + 12x + 11$   
 $-11 = x^2 + 12x + 36$  (PST)  $\left(\frac{12}{2}\right)^2$   
 $+36$   
 $25 = x^2 + 12x + 36$   
 $25 = (x+6)(x+6)$   
 $\pm\sqrt{25} = \sqrt{(x+6)^2}$   
 $\pm 5 = x+6$   
 $-6$   
 $-1, -11 = x$

6.  $27 = 3x^2 + 12x$   $\div 3$   
 $9 = x^2 + 4x + 4$  (PST)  $\left(\frac{4}{2}\right)^2$   
 $+4$   
 $13 = (x+2)^2$   
 $\pm\sqrt{13} = \sqrt{(x+2)^2}$   
 $\pm\sqrt{13} = x+2$   
 $-2$   
 $-2 \pm \sqrt{13} = x$

7.  $0 = 2x^2 + 6x - 14$   $\div 2$   
 $0 = x^2 + 3x - 7$   
 $+7$   
 $7 = x^2 + 3x + \frac{9}{4}$  (PST)  $\left(\frac{3}{2}\right)^2$   
 $+ \frac{9}{4}$   
 $7 + \frac{9}{4} = (x + \frac{3}{2})^2$   
 $\frac{28}{4} + \frac{9}{4} = \frac{37}{4} = (x + \frac{3}{2})^2$   
 $\pm\sqrt{\frac{37}{4}} = x + \frac{3}{2}$   
 $\pm\frac{\sqrt{37}}{2} = x + \frac{3}{2}$   
 $-\frac{3}{2} \pm \frac{\sqrt{37}}{2} = x$   
 or  
 $-\frac{3 \pm \sqrt{37}}{2} = x$

Write the equation in vertex form, and identify the maximum or minimum value of the graph of the function.

8.  $y = x^2 + 6x - 6$   
 $y = x^2 + 6x + 9 - 6 - 9$   
 $y = (x+3)^2 - 15$  (PST)  $\left(\frac{6}{2}\right)^2$   
 $a > 0$   
 minimum:  $-15$

9.  $y = -2x^2 + 20x - 42$   
 $y = -2(x^2 - 10x + 25) - 42 + 2(25)$   
 $y = -2(x-5)^2 + 8$  (PST)  $\left(\frac{-10}{2}\right)^2$   
 $a < 0$   
 maximum:  $8$

10. The daily profit for a company is modeled by the function  $p(x) = -0.5x^2 + 40x - 300$ , where  $x$  is the number of units sold. How many units does the company need to sell each day to maximize profits?

