Activity

2-6 The Quadratic Formula

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EXPLORE & REASON

PROOF

You can complete the square to solve the general quadratic equation, $ax^2 + bx + c = 0$.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \left(\frac{b}{a}\right)x = -\frac{c}{a}$$

$$x^2 + \left(\frac{b}{a}\right)x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Completed the Square

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

A. **Construct Arguments** Justify each step in this general solution. MP.3

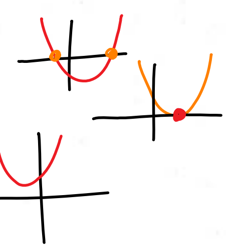
$i = \sqrt{-1}$

discriminant

if $b^2 - 4ac \overset{\text{pos}}{>} 0$, then 2 real solns

" " $\overset{\text{zero}}{=} 0$, " 1 " "

" " $\overset{\text{neg}}{<} 0$, " 0 " " $\rightarrow$ 2 imaginary solutions



B. What must be true of the value of $b^2 - 4ac$ if the equation $ax^2 + bx + c = 0$ has two non-real solutions? If it has just one solution?

negative

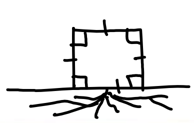
zero

$$\frac{\ldots \pm\sqrt{0}}{\ldots}$$

HABITS OF MIND

Communicate Precisely Why is there a $\pm$ in the second to last step of the derivation of the Quadratic Formula? MP.6

$\rightarrow$ square root



Notes

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

EXAMPLE 1

Try It! Solve Quadratic Equations

1. Solve using the Quadratic Formula.

a. $2x^2 + 6x + 3 = 0$

$a=2 \quad b=6 \quad c=3$

$x = \dfrac{-(6) \pm \sqrt{(6)^2 - 4(2)(3)}}{2(2)}$

$x = \dfrac{-6 \pm \sqrt{36 - 24}}{4}$

$x = \dfrac{-6 \pm \sqrt{12}}{4} = \dfrac{-6 \pm 2\sqrt{3}}{4}$

$x = \dfrac{\cancel{2}(-3 \pm 1\sqrt{3})}{\cancel{4}\,2} = \boxed{\dfrac{-3 \pm \sqrt{3}}{2}}$ or $\boxed{\dfrac{-3}{2} \pm \dfrac{\sqrt{3}}{2}}$

b. $3x^2 - 2x + 7 = 0$

$a:3 \quad b:-2 \quad c:7$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(7)}}{2(3)}$

$= \dfrac{2 \pm \sqrt{4 - 84}}{6} = \dfrac{2 \pm \sqrt{-80}}{6}$

$-1, 2, 2, 2, 2, 5$

$= \dfrac{2 \pm 4i\sqrt{5}}{6} = \dfrac{\cancel{2}(1 \pm 2i\sqrt{5})}{\cancel{6}\,3}$

$= \boxed{\dfrac{1 + 2\sqrt{5}\,i}{3}}$ or $\boxed{\dfrac{1}{3} \pm \dfrac{2\sqrt{5}}{3}i}$

EXAMPLE 2

Try It! Choose a Solution Method

2. Solve the equation $6x^2 + x - 15 = 0$ using the Quadratic Formula and one other method.

$a=6 \quad b=1 \quad c=-15$

$x = \dfrac{-(1) \pm \sqrt{(1)^2 - 4(6)(-15)}}{2(6)}$

$= \dfrac{-1 \pm \sqrt{1 + 360}}{12} = \dfrac{-1 \pm \sqrt{361}}{12}$

$= \dfrac{-1 \pm 19}{12} = \dfrac{-1+19}{12}$ or $\dfrac{-1-19}{12}$

$= \dfrac{18}{12}$ or $\dfrac{-20}{12}$

$= \dfrac{3}{2}$ or $\dfrac{-5}{3}$

1, 10, -9, -90

	$3x$	$+5$
$2x$	$6x^2$	$10x$
-3	$-9x$	-15

$(3x+5)(2x-3) = 0$

$x = \dfrac{-5}{3} \qquad x = \dfrac{3}{2}$

HABITS OF MIND

Construct Arguments Is it possible for a quadratic equation to have one real solution and one complex solution? Explain. MP.3

No.... Complex #s come in pairs

EXAMPLE 3

Try It! Identify the Number of Real-Number Solutions

discriminant
$b^2 - 4ac$

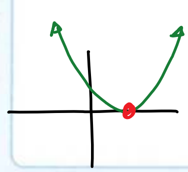
3. Describe the nature of the solutions for each equation.

a. $16x^2 + 8x + 1 = 0$

$(8)^2 - 4(16)(1)$

$\to 64 - 64 \to 0$

1 real soln.



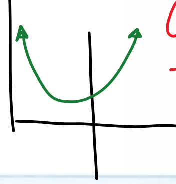
b. $2x^2 - 5x + 6 = 0$

$(-5)^2 - 4(2)(6)$

$\to 25 - 48 \to -23$

0 reals

$\to$ 2 imag.

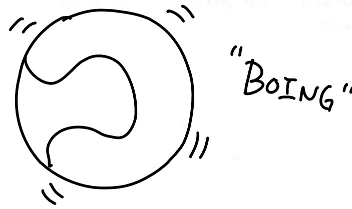


Notes

Assess

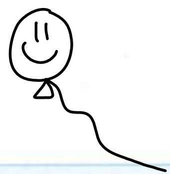
EXAMPLE 4 **Try It!** **Interpret the Discriminant**

4. According to the model of Rachel's serve, will the ball reach a height of 3 meters?



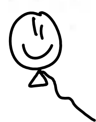
HABITS OF MIND

Reason Create a quadratic equation that has two complex solutions. **MP.2**



EXAMPLE 5 **Try It!** **Use the Discriminant to Find a Particular Equation**

5. Determine the value(s) of b that ensure $5x^2 + bx + 5 = 0$ has two non-real solutions.



HABITS OF MIND

Use Appropriate Tools Why is the Quadratic Formula helpful? **MP.5**

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you use the Quadratic Formula to solve quadratic equations or to predict the nature of their solutions?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. **Vocabulary** Why is the discriminant a useful tool to use when solving quadratic equations?

3. **Error Analysis** Rick claims that the equation $x^2 + 5x + 9 = 0$ has no solution. Jenny claims that there are two solutions. Explain how Rick could be correct, and explain how Jenny could be correct. MP.3

4. **Use Appropriate Tools** What methods can you use to solve quadratic equations? MP.5

Do You KNOW HOW?

5. Describe the number and type of solutions of the equation $2x^2 + 7x + 11 = 0$.

$a=2$ $b=7$ $c=11$

$b^2 - 4ac$

$(7)^2 - 4(2)(11)$

$= 49 - 88 = -39$

no real solutions

$\rightarrow$ 2 imag solns

6. Use the Quadratic Formula to solve the equation $x^2 + 6x - 10 = 0$.

$a=1$ $b=6$ $c=-10$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-10)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36+40}}{2} = \frac{-6 \pm \sqrt{76}}{2}$$

2, 2, 19

$$= \frac{-6 \pm 2\sqrt{19}}{2} = \boxed{-3 \pm \sqrt{19}}$$

7. At time t seconds, the height, h, of a ball thrown vertically upward is modeled by the equation $h = -5t^2 + 33t + 4$. About how long will it take for the ball to hit the ground?

GC
- graph $\rightarrow$ roots
- equa $\rightarrow$ poly $\rightarrow$ deg 2

$x = \boxed{6.72 \text{ secs}}$

8. Use the Quadratic Formula to solve the equation $x^2 - 8x + 16 = 0$. Is this the only way to solve this equation? Explain.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{64-64}}{2} = \frac{8 \pm \sqrt{0}}{2}$$

$= 4$

PST:

$(x-4)(x-4)=0$

$x-4=0$ or $x-4=0$

$x=4$