Activity

3-1 Graphing Polynomial Functions

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EXPLORE & REASON

Consider functions of the form $f(x) = x^n$, where n is a positive integer.

A. Graph $f(x) = x^n$ for $n = 1$, 3, and 5. Look at the graphs in Quadrant I. As the exponent increases, what is happening to the graphs? Which quadrants do the graphs pass through?

B. **Look for Relationships** Now graph $f(x) = x^n$ for $n = 2$, 4, and 6. What happens to these graphs in Quadrant I as the exponent increases? Which quadrants do the graphs pass through? MP.7

C. Write two equations in the form $f(x) = x^n$ with graphs that you predict are in Quadrants I and II. Write two equations with graphs that you predict are in Quadrants I and III. Use graphing technology to test your predictions.

HABITS OF MIND

Construct Arguments Compare and contrast the end behavior of the graphs of $f(x) = x^n$ when $n = 1$, 3, & 5 with the graphs of $f(x) = x^n$ when $n = 2$, 4, & 10. Write a general statement that compares the end behavior of the graphs when the exponents are odd to the end behavior when the exponents are even. MP.3

EXAMPLE 1 **Try It! Classify Polynomials**

1. What is each polynomial in standard form. What are the leading coefficient, the degree, and the number of terms of each?
   a. $2x - 3x^4 + 6 - 5x^3$
   b. $x^5 + 2x^6 - 3x^4 - 8x + 4x^3$

EXAMPLE 2 **Try It! Interpret Leading Coefficients and Degrees**

2. Use the leading coefficient and degree of the polynomial function to determine the end behavior of each graph.
   a. $f(x) = 2x^6 - 5x^5 + 6x^4 - x^3 + 4x^2 - x + 1$

   b. $g(x) = -5x^3 + 8x + 4$

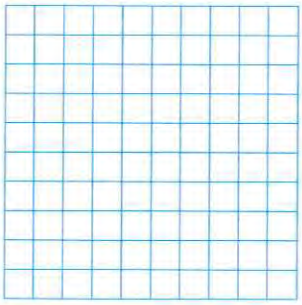
HABITS OF MIND

Communicate Precisely How does the leading coefficient help determine the end behavior of an even function? MP.6

EXAMPLE 3 **Try It! Graph a Polynomial Function**

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.
   a. Make a table of values to identify key features and sketch a graph of the function.

   b. Find the average rate of change over the interval [0, 2].

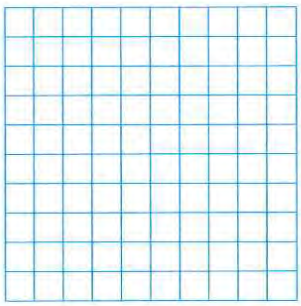


Notes

Assess

EXAMPLE 4 Try It! Sketch a Graph from a Verbal Description

4. Use the information below to sketch a graph of the polynomial function $y = f(x)$.
   - $f(x)$ is positive on the intervals $(-2, -1)$ and $(1, 2)$.
   - $f(x)$ is negative on the intervals $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$.
   - $f(x)$ is increasing on the intervals $(-\infty, -1.5)$ and $(0, 1.5)$.
   - $f(x)$ is decreasing on the intervals $(-1.5, 0)$ and $(1.5, \infty)$.



HABITS OF MIND

Generalize What can you tell about the graph of a function if its equation has an odd degree and a negative leading coefficient? MP.8

EXAMPLE 5 Try It! Interpret a Polynomial Model

5. Danielle is engineering a new brand of shoes. For x shoes sold, in thousands, a revenue of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$ dollars, in ten thousands, will be earned.
   a. How much will be earned in revenue for selling 1,000 shoes?

   b. What do the x- and y-intercepts of the graph mean in this context? Do those values make sense?

HABITS OF MIND

Use Appropriate Tools Estimate the turning point of the graph of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$. What does this point represent in the context of Try It! 5? MP.5

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do the key features of a polynomial function help you sketch its graph?

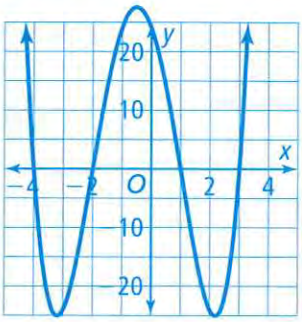
2. **Error Analysis** Allie said the degree of the polynomial function $f(x) = x^5 + 2x^4 + 3x^3 - 2x^6 - 9x^2 - 6x + 4$ is 5. Explain and correct Allie's error. **MP.3**

3. **Vocabulary** Explain how to determine the **leading coefficient** of a polynomial function.

4. **Look for Relationships** What is the relationship between the degree and leading coefficient of a polynomial function and the end behavior of the polynomial? **MP.7**

Do You KNOW HOW?

The graph shows the function $f(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$. Find the following.



5. number of terms

6. degree

7. leading coefficient

8. end behavior

9. turning point(s)

10. x-intercept(s)

11. relative minimum(s)

12. relative maximum(s)