Polynomials

3-1 Graphing Polynomial Functions

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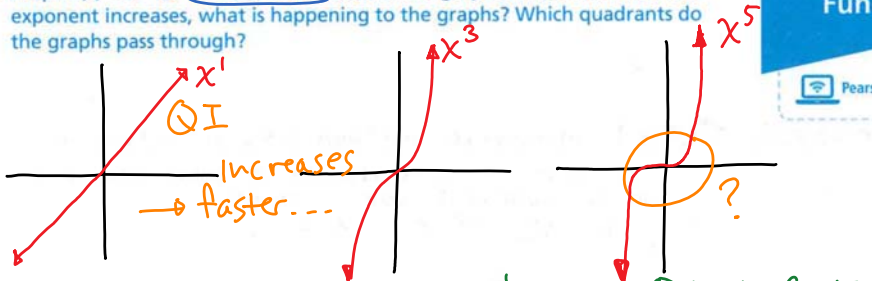
EXPLORE & REASON

Consider functions of the form $f(x) = x^n$, where n is a positive integer.

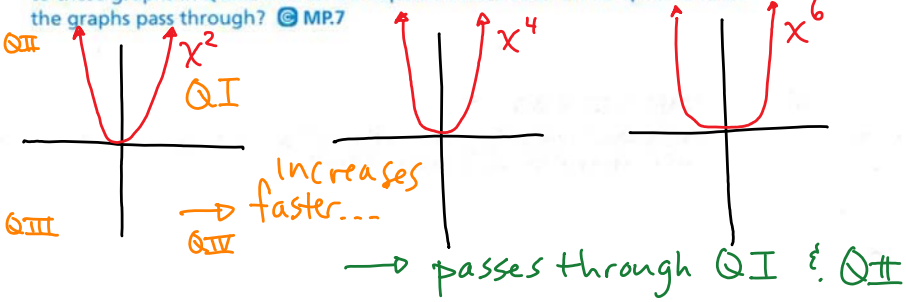
odd functions

A. Graph $f(x) = x^n$ for $n = 1, 3$, and 5. Look at the graphs in Quadrant I. As the exponent increases, what is happening to the graphs? Which quadrants do the graphs pass through?



even functions

B. Look for Relationships Now graph $f(x) = x^n$ for $n = 2, 4$, and 6. What happens to these graphs in Quadrant I as the exponent increases? Which quadrants do the graphs pass through? MP.7



C. Write two equations in the form $f(x) = x^n$ with graphs that you predict are in Quadrants I and II. Write two equations with graphs that you predict are in Quadrants I and III. Use graphing technology to test your predictions.

degree: linear, quadratic, cubic, quartic, quintic

Classify Polynomials

$-4x + 9 + 2x^3$

mono, bi, tri, ... (1, 2, 3)

standard form
- powers in descending order...

$2x^3 - 4x^1 + 9x^0$

leading coefficient

degree of the polynomial
- highest degree of each monomial

→ 3rd degree / cubic

→ 3 terms / trinomial

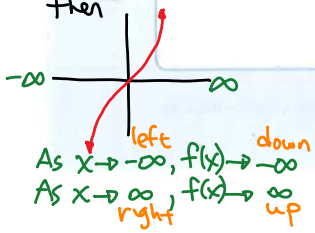
HABITS OF MIND

Construct Arguments Compare and contrast the end behavior of the graphs of $f(x) = x^n$ when $n = 1, 3$, & 5 with the graphs of $f(x) = x^n$ when $n = 2, 4$, & 10. Write a general statement that compares the end behavior of the graphs when the exponents are odd to the end behavior when the exponents are even. MP.3

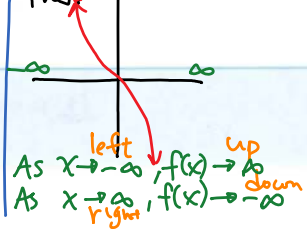
Polynomial

$f(x) = ax^n$

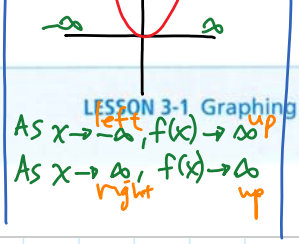
If n = odd & a > 0 (pos), then



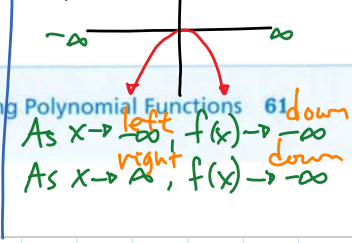
If n = odd & a < 0 (neg), then



If n = even & a > 0 (pos), then



If n = even & a < 0 (neg) then



Notes

Assess

EXAMPLE 1 Try It! Classify Polynomials

1. What is each polynomial in standard form. What are the leading coefficient, the degree, and the number of terms of each?

a. $2x - 3x^4 + 6 - 5x^3$ → $-3x^4 - 5x^3 + 2x + 6$ — leading coefficient: -3; degree: 4; 4 terms

b. $x^5 + 2x^6 - 3x^4 - 8x + 4x^3$ → $2x^6 + x^5 - 3x^4 + 4x^3 - 8x$ — leading coef: 2; degree: 6; 5 terms

EXAMPLE 2 Try It! Interpret Leading Coefficients and Degrees

ax^n: even — $a>0$: up, up; $a<0$: down, down

ax^n: odd — $a>0$: down, up; $a<0$: up, down

2. Use the leading coefficient and degree of the polynomial function to determine the end behavior of each graph.

a. $f(x) = 2x^6 - 5x^5 + 6x^4 - x^3 + 4x^2 - x + 1$ — $a>0$; even

As $x \to -\infty$ (left), $f(x) \to \infty$ (up)

As $x \to \infty$ (right), $f(x) \to \infty$ (up)

b. $g(x) = -5x^3 + 8x + 4$ — $a<0$; odd

As $x \to -\infty$ (left), $f(x) \to +\infty$ (up)

As $x \to +\infty$ (right), $f(x) \to -\infty$ (down)

HABITS OF MIND

Communicate Precisely How does the leading coefficient help determine the end behavior of an even function? MP.6

neg? vert. refl.

EXAMPLE 3 Try It! Graph a Polynomial Function

$f(x) = -\frac{1}{2}x^4 + 3x^2 + 2$

3. Consider the polynomial function $f(x) = x^5 + 16x^2 + 10x + 11$

a. Make a table of values to identify key features and sketch a graph of the function.

- even
- $a<0$
- degree 4 → 3 Turning Points / U-Turns

b. Find the average rate of change over the interval $[-2, 0]$

slope: $\frac{\text{rise}}{\text{run}} = \frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$ — y values; a, b; x-values; delta: change

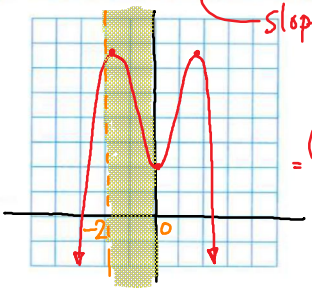
$$= \frac{\left(-\frac{1}{2}(0)^4 + 3(0)^2 + 2\right) - \left(-\frac{1}{2}(-2)^4 + 3(-2)^2 + 2\right)}{0 - (-2)}$$

$$= \frac{2 - (-8 + 12 + 2)}{2} = \frac{2-(6)}{2}$$

$$= -\frac{4}{2} = -2$$

relative max: (-1.73, 6.5), (1.73, 6.5)

relative min: (0, 2)



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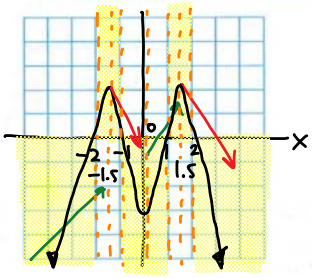
Notes

Assess

EXAMPLE 4 **Try It! Sketch a Graph from a Verbal Description**

4. Use the information below to sketch a graph of the polynomial function $y = f(x)$.
   - $f(x)$ is positive on the intervals $(-2, -1)$ and $(1, 2)$.
   - $f(x)$ is negative on the intervals $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$.
   - $f(x)$ is increasing on the intervals $(-\infty, -1.5)$ and $(0, 1.5)$.
   - $f(x)$ is decreasing on the intervals $(-1.5, 0)$ and $(1.5, \infty)$.



HABITS OF MIND

Generalize What can you tell about the graph of a function if its equation has an odd degree and a negative leading coefficient? MP.8

EXAMPLE 5 **Try It! Interpret a Polynomial Model**

5. Danielle is engineering a new brand of shoes. For x shoes sold, in thousands, a revenue of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$ dollars, in ten thousands, will be earned.
   a. How much will be earned in revenue for selling 1,000 shoes?

   b. What do the x- and y-intercepts of the graph mean in this context? Do those values make sense?

HABITS OF MIND

Use Appropriate Tools Estimate the turning point of the graph of $p(x) = -3x^4 + 4x^3 - 2x^2 + 5x + 10$. What does this point represent in the context of Try It! 5? MP.5

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How do the key features of a polynomial function help you sketch its graph?

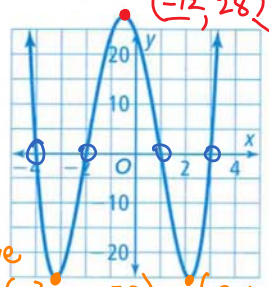
2. **Error Analysis** Allie said the degree of the polynomial function $f(x) = x^5 + 2x^4 + 3x^3 - 2x^6 - 9x^2 - 6x + 4$ is 5. Explain and correct Allie's error. MP.3

3. **Vocabulary** Explain how to determine the **leading coefficient** of a polynomial function.

4. **Look for Relationships** What is the relationship between the degree and leading coefficient of a polynomial function and the end behavior of the polynomial? MP.7

Do You KNOW HOW?

The graph shows the function $f(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$. Find the following.



(-½, 28) relative max: 28

relative min (-3.2, -25.3) (2.126, -25.3)

5. number of terms

5 terms

6. degree

4

7. leading coefficient

1

8. end behavior

As $x \to -\infty$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to \infty$

9. turning point(s)

3

10. x-intercept(s)

$-4, -2, 1, 3$

11. relative minimum(s)

see above

12. relative maximum(s)

see above

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