Activity

3-3 Polynomial Identities

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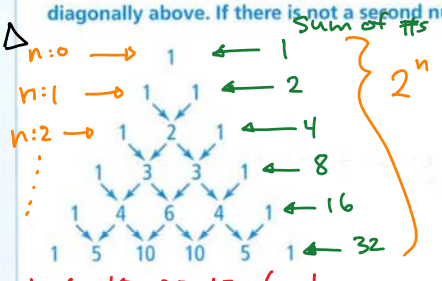
EXPLORE & REASON

Look at the following triangle. Each number is the sum of the two numbers diagonally above. If there is not a second number, think of it as 0.

Pascal's Δ



1 6 15 20 15 6 1

Linked to Binomial Expansions... $(a+b)^n$

A. Write the numbers in the next 3 rows.

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

Polynomial Identities

- Difference of Squares: $a^2-b^2=(a-b)(a+b)$ ÷ Product of a Sum & diff. $(a+b)(a+b)$
- Square of a Sum: $(a+b)^2=a^2+2ab+b^2$ × Perfect Square Trinomial

B. Look for Relationships What other patterns do you see? MP.7

- Symmetry
- Difference of Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$
- Sum of Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

C. Write a formula for the sum of the numbers in the n^{th} row of the triangle. Find the sum of each row:

2^n = 1, 2, 4, 8, 16, 32, ...

HABITS OF MIND

Look for Relationships Create a triangle that starts with 2 instead of 1. How does this new triangle relate to the original triangle? MP.7

Diff. of Cubes & Sum of Cubes

Diff of Cubes & Sum of Cubes

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

Notes

Assess

EXAMPLE 1 **Try It! Prove a Polynomial Identity**

1. Prove the Difference of Cubes Identity.

$(a-b)(a^2+ab+b^2)$

$a^3+a^2b+ab^2-a^2b-ab^2-b^3 \rightarrow a^3-b^3$

HABITS OF MIND

Reason Is the trinomial factor in the Difference of Cubes Identity a perfect square trinomial? Explain. MP.2

EXAMPLE 2 **Try It! Use Polynomial Identities to Multiply Polynomials**

2. Use polynomial identities to multiply each expression.

a. $(3x^2+5y^3)(3x^2-5y^3)$ — diff of squares

$(a+b)(a-b) \rightarrow a^2-b^2 \rightarrow (3x^2)^2-(5y^3)^2$

• subst a w/ $3x^2$ & b w/ $5y^3$ $\rightarrow 9x^4-25y^6$

Product of a Sum & diff

b. $(12+15)^2$ — PST — Product of a Sum

$(a+b)^2 \rightarrow a^2+2ab+b^2 \rightarrow (12)^2+2(12)(15)+(15)^2$

• subst a w/ 12 & b w/ 15

$\rightarrow 144+360+225$

$\rightarrow 729$

FOIL ?

EXAMPLE 3 **Try It! Use Polynomial Identities to Factor Polynomials**

3. Use polynomial identities to factor each polynomial or simplify each expression.

$a^2-b^2=(a-b)(a+b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^3+b^3=(a+b)(a^2-ab+b^3)$

a. m^8-9n^{10} — diff of squares → ½ of power — a − b, a + b

$(m^4)^2-(3n^5)^2 \rightarrow (m^4-3n^5)(m^4+3n^5)$

b. $27x^9-343y^6$ — diff of cubes → ⅓ of power — a, b

$(3x^3)^3-(7y^2)^3 \rightarrow ((3x^3)-(7y^2))((3x^3)^2+(3x^3)(7y^2)+(7y^2)^2)$

$\rightarrow (3x^3-7y^2)(9x^6+21x^3y^2+49y^4)$

c. 12^3+2^3 — sum of cubes — a, b

$(12)^3+(2)^3 \rightarrow ((12)+(2))((12)^2-(12)(2)+(2)^2)$

$\rightarrow (14)(144-24+4) \rightarrow (14)(124)$

$\rightarrow 1736$

HABITS OF MIND

Look for Relationships What binomial has factors $(a-3b)$ and $(a^2+3ab+9b^2)$? MP.7

↑ FOIL or Pattern? "a" "b"

→ diff of cubes

"a"^{3} − "b"^{3} → $a^3-(3b)^3$

→ a^3-27b^3

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Notes

Assess

Row 0: 1
1: 1 1
2: 1 2 1
3: 1 3 3 1
4: 1 4 6 4 1
5: 1 5 10 10 5 1
6: 1 6 15 20 15 6 1
7: 1 7 21 35 35 21 7 1

EXAMPLE 4 **Try It! Expand a Power of a Binomial**

4. Use Pascal's Triangle to expand $(x+y)^6$.

"a" power decreases from n

"b" power increases from 0

$1x^6y^0+6x^5y^1+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+1y^6$

$\rightarrow x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6$

Binomial Theorem: $(a+b)^n = C_0a^n + C_1a^{n-1}b + C_2a^{n-2}b^2 + \ldots + C_{n-1}ab^{n-1} + C_nb^n$

EXAMPLE 5 **Try It! Apply the Binomial Theorem**

5. Use the Binomial Theorem to expand each expression.

a. $(x-1)^7$

$a=x \quad b=-1$

$\rightarrow 1x^7 + 7x^6(-1) + 21x^5(-1)^2 + 35x^4(-1)^3 + 35x^3(-1)^4 + 21x^2(-1)^5 + 7x(-1)^6 + 1(-1)^7$

$\rightarrow x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$

1
3 2
6
2

b. $(2c+d)^6$

$a=2c \quad b=d$

$\rightarrow 1(2c)^6 + 6(2c)^5d + 15(2c)^4d^2 + 20(2c)^3d^3 + 15(2c)^2d^4 + 6(2c)d^5 + 1d^6$

$\rightarrow 64c^6 + 192c^5d + 240c^4d^2 + 160c^3d^3 + 60c^2d^4 + 12cd^5 + d^6$

HABITS OF MIND

Use Structure For what binomial expression is the expansion $243x^5 - 405x^4y^2 + 270x^3y^4 - 90x^2y^6 + 15xy^8 - y^{10}$? MP.7

3^5

$(3x - y^2)^5$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you use polynomial identities to rewrite expressions efficiently?

2. **Reason** Explain why the middle term of $(x + 5)^2$ is $10x$. MP.2

3. **Communicate Precisely** How are Pascal's Triangle and a binomial expansion, such as $(a + b)^5$, related? MP.6

4. **Use Structure** Explain how to use a polynomial identity to factor $8x^6 - 27y^3$. MP.7

5. **Make Sense and Persevere** What does C_3 represent in the expansion $C_0a^5 + C_1a^4b + C_2a^3b^2 + C_3a^2b^3 + C_4ab^4 + C_5b^5$? MP.1

6. **Error Analysis** Dakota said the third term of the expansion of $(2g + 3h)^4$ is $54gh^3$. Explain Dakota's error. Then correct the error. MP.3

Do You KNOW HOW?

Use polynomial identities to multiply each expression.

7. $(2x + 8y)(2x - 8y)$ — prod sum & diff — FOIL: $(2x)^2 - (8y)^2$, $4x^2 - 64y^2$ — diff of squares

8. $(x + 3y^3)^2$ — $(x+3y^3)(x+3y^3) \rightarrow x^2 + 6xy^3 + 9y^6$ — prod of a sum — PST

Use polynomial identities to factor each polynomial.

9. $36a^6 - 4b^2$ — mono GCF: $4(9a^6 - b^2)$ ...

10. $8x^6 - y^3$ — diff of cubes — $a = 2x^2$, $b = y$; $a^3 - b^3 = (a-b)(a^2+ab+b^2)$; $= (2x^2 - y)((2x^2)^2 + (2x^2)y + (y)^2)$; $= (2x^2 - y)(4x^4 + 2x^2y + y^2)$

11. $m^9 + 27n^6$ — sum of cubes

Find the term of each binomial expansion.

12. fifth term of $(x + y)^5$ ← 6 terms — a b — 1 5 10 10 5 1 — $\rightarrow 5xy^4$

13. third term of $(a - 3)^6$ — 1 6 15 — $\rightarrow 135a^4$

Use Pascal's Triangle to expand each expression.

14. $(x + 1)^5$

15. $(a - b)^6$

Use the Binomial Theorem to expand each expression.

16. $(d - 1)^4$

17. $(x + y)^7$