3-4 Dividing Polynomials

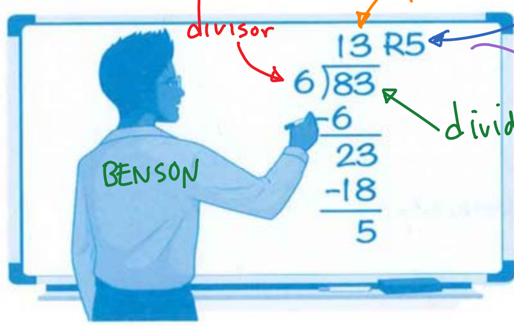
EXPLORE & REASON

Benson recalls how to divide whole numbers by solving a problem with 6 as the divisor and 83 as the dividend. He determines that the quotient is 13 with remainder 5.

quotient

remainder

divisor



dividend

$83 \div 6 \rightarrow \frac{83}{6}$

$\rightarrow 13\frac{5}{6}$ or $\boxed{13 + \frac{5}{6}}$

remainder

divisor

quotient

A. Explain the process of long division using Benson's example.

see above ... "How many times can the divisor divide into the dividend?"

B. How can you express the remainder as a fraction?

$\frac{5}{6}$

C. **Use Structure** Use the results of the division problem to write two expressions for 83 that include the divisor, quotient, and remainder. MP.7

$13\frac{5}{6}$ or $13 + \frac{5}{6}$

HABITS OF MIND

Look for Relationships If the remainder in a division problem is zero, what can you say about the dividend? MP.7

→ divisible

$$\begin{array}{r} 13 \\ 6\overline{)\,83} \\ (-)\;6 \\ \hline 23 \\ (-)\;18 \\ \hline R\;5 \end{array}$$

Notes

Assess

EXAMPLE 1 **Try It!** **Use Long Division to Divide Polynomials**

1. Use long division to divide the polynomials.

a. $\underbrace{x^3 - 6x^2 + 11x - 6}_{\text{dividend}}$ divided by $\underbrace{x^2 - 4x + 3}_{\text{divisor}}$

$$\begin{array}{r} x-2 \\ x^2-4x+3\overline{)\,x^3-6x^2+11x-6} \\ (-)\;x^3-4x^2+3x \\ \hline -2x^2+8x-6 \\ (-)\;-2x^2+8x-6 \\ \hline R\;\;0 \end{array} \Rightarrow \boxed{x-2}$$

b. $16x^4 - 85$ divided by $4x^2 + 9$

$$\begin{array}{r} 4x^2 \qquad -9 \\ 4x^2+9\overline{)\,16x^4+0x^3+0x^2+0x-85} \\ (-)\;16x^4 \qquad 36x^2 \qquad\qquad \\ \hline -36x^2 \qquad -85 \\ (-)\;-36x^2 \qquad -81 \\ \hline R\;\;-4 \end{array} \Rightarrow \boxed{4x^2-9+\frac{-4}{4x^2+9}}$$

EXAMPLE 2 **Try It!** **Use Synthetic Division to Divide by $\boxed{x - a}$**

$0x^2$: placeholder

a: root/x-int/solution/zero

2. Use synthetic division to divide $\underbrace{3x^3 - 5x + 10}_{\text{coefficients}}$ by $x - 1$.

a: 1

$$\begin{array}{r|rrrr} 1 & 3 & 0 & -5 & 10 \\ & & 3 & 3 & -2 \\ \hline & 3 & 3 & -2 & \boxed{8} \\ & x^2 & x & c & \text{remainder} \end{array}$$

- add vertically
- mult diagonally

$\Rightarrow 3x^2+3x-2+\frac{8}{x-1}$

ex2) $2x^3-7x^2-4$ divided by $x-3$.

$x-a$

a: 3

$$\begin{array}{r|rrrr} 3 & 2 & -7 & 0 & -4 \\ & & 6 & -3 & -9 \\ \hline & 2 & -1 & -3 & \boxed{-13} \\ & x^2 & x & c & r \end{array}$$

$\Rightarrow 2x^2-x-3+\frac{-13}{x-3}$

HABITS OF MIND

Communicate Precisely Which method would you use to divide a polynomial by $\underbrace{x^2 + 5}_{\text{not } x-a}$? Why? MP.6

$\therefore$: Long $\div$

Remainder Theorem

If $p(x)$ is $\div$ by $x-a$, then the remainder is $P(a)$

Subst "a" into the polynomial

EXAMPLE 3 **Try It!** **Relate $P(a)$ to the Remainder of $P(x) \div (x - a)$**

3. Use synthetic division to show that the remainder of $f(x) = x^3 + 8x^2 + 12x + 5$ divided by $x + 2$ is equal to $f(-2)$.

$$\underbrace{\begin{array}{r|rrrr} -2 & 1 & 8 & 12 & 5 \\ & & -2 & -12 & 0 \\ \hline & 1 & 6 & 0 & \boxed{5} \end{array}}_{\text{"fast"}}$$

$f(-2) =$

$(-2)^3+8(-2)^2+12(-2)+5$

$\rightarrow -8+32-24+5$

$\rightarrow 5$

$f(a)$

$(x+2) \leftarrow$ divisor

traditional

Factor Theorem

The expression $x-a$ is a factor of $P(x)$ iff $P(a)=0$

if and only if

remainder 0?

Notes

Assess

EXAMPLE 4 ## Try It! Use the Remainder Theorem to Evaluate Polynomials

4. A technology company uses the function $R(x) = -x^3 + 12x^2 + 6x + 80$ to model expected annual revenue, in thousands of dollars, for a new product, where x is the number of years after the product is released. Use the Remainder Theorem to estimate the revenue in year 5.

$\$$

+ x

$$\begin{array}{r|rrrr} 5 & -1 & 12 & 6 & 80 \\ & & -5 & 35 & 205 \\ \hline & -1 & 7 & 41 & \boxed{285} \end{array}$$

→ $285,000

EXAMPLE 5 ## Try It! Use the Factor Theorem

5. Use the Remainder and Factor Theorems to determine whether the given binomial is a factor of $P(x)$.

$x-4$

a. $P(x) = x^3 - 10x^2 + 28x - 16$; binomial: $x - 4$

a:4

+ x

$$\begin{array}{r|rrrr} 4 & 1 & -10 & 28 & -16 \\ & & 4 & -24 & 16 \\ \hline & 1 & -6 & 4 & \boxed{0} \end{array}$$

R

→ $(x-4)$ is a factor of $P(x)$

& $P(4)=0$

b. $P(x) = 2x^4 + 9x^3 - 2x^2 + 6x - 40$; binomial: $x + 5$

a: −5

+ x

$$\begin{array}{r|rrrrr} -5 & 2 & 9 & -2 & 6 & -40 \\ & & -10 & 5 & -15 & 45 \\ \hline & 2 & -1 & 3 & -9 & \boxed{5} \\ & x^3 & x^2 & x & c & R \end{array}$$

→ $(x+5)$ is not a factor of $P(x)$

because $P(-5)=5$

HABITS OF MIND

Make Sense and Persevere Is $x - 2$ a factor of $x^5 + x^4 - 6x^3 + 2x^2 - 11x + 15$? If not, what is the remainder? MP.1

$$\begin{array}{r|rrrrrr} 2 & 1 & 1 & -6 & 2 & -11 & 15 \\ & & 2 & 6 & 0 & 4 & -14 \\ \hline & 1 & 3 & 0 & 2 & -7 & \boxed{1} \end{array}$$

remainder

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you divide polynomials?

$\frac{x^4}{x^2}$

$\frac{-4x^3}{x^2}$

$-3x - 32$

$\frac{4x^2}{x^2}$

2. **Error Analysis** Ella said the remainder of $x^3 + 2x^2 - 4x + 6$ divided by $x + 5$ is 149. Is Ella correct? Explain. MP.3

3. **Look for Relationships** You divide a polynomial $P(x)$ by a linear expression $D(x)$. You find a quotient $Q(x)$ and a remainder $R(x)$. How can you check your work? MP.7

Do You KNOW HOW?

4. Use long division to divide $x^4 - 4x^3 + 12x^2 - 3x + 6$ by $x^2 + 8$.

divisor

dividend

$$
\begin{array}{r}
x^2 - 4x + 4 \\
x^2+8 \,\big)\, x^4 - 4x^3 + 12x^2 - 3x + 6 \\
(-)\; x^4 \qquad\quad + 8x^2 \qquad\qquad \\
\hline
-4x^3 + 4x^2 - 3x \qquad \\
(-)\; -4x^3 \qquad - 32x \qquad \\
\hline
4x^2 + 29x + 6 \\
(-)\; 4x^2 \qquad\quad + 32 \\
\hline
R \quad 29x - 26
\end{array}
$$

$$\boxed{x^2 - 4x + 4 + \frac{29x - 26}{x^2 + 8}}$$

5. Use synthetic division to divide $x^3 - 8x^2 + 9x - 5$ by $x - 3$. $a: 3$

+ x

$$
\begin{array}{c|cccc}
3 & 1 & -8 & 9 & -5 \\
 & & 3 & -15 & -18 \\
\hline
 & 1 & -5 & -6 & \boxed{-23} \\
 & x^2 & x & c & R
\end{array}
$$

$$\boxed{x^2 - 5x - 6 + \frac{-23}{x - 3}}$$

$x - a$

$a: -2$

6. Use the Remainder Theorem to find the remainder of $2x^4 + x^2 - 10x - 1$ divided by $x + 2$. → synth ÷

$$
\begin{array}{c|ccccc}
-2 & 2 & 0 & 1 & -10 & -1 \\
 & & -4 & 8 & -18 & 56 \\
\hline
 & 2 & -4 & 9 & -28 & 55
\end{array}
$$

remainder

$x - a$ $\quad a: -9$

7. Is $x + 9$ a factor of the polynomial $P(x) = x^3 + 11x^2 + 15x - 27$? If so, write the polynomial as a product of two factors. If not, explain how you know.

$$
\begin{array}{c|cccc}
-9 & 1 & 11 & 15 & -27 \\
 & & -9 & -18 & 27 \\
\hline
 & 1 & 2 & -3 & \boxed{0} \\
 & x^2 & x & c & R
\end{array}
$$

$$\boxed{(x^2 + 2x - 3)(x + 9)}$$

can be broken down / factored further

$(x+3)(x-1)(x+9)$