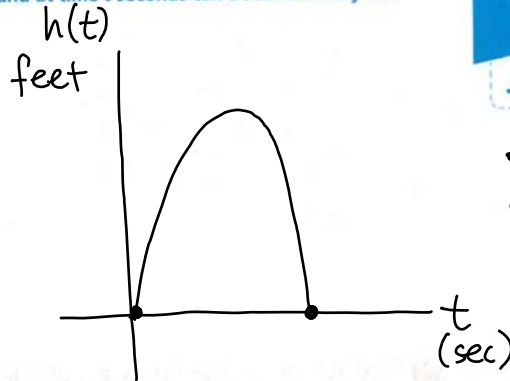
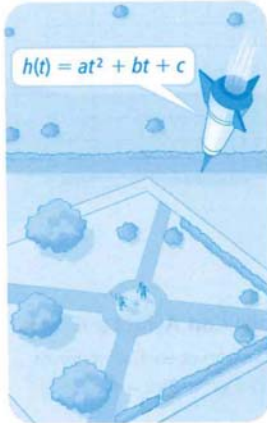


MODEL & DISCUSS

Charlie and Aisha built a small rocket and launched it from their backyard. The rocket fell to the ground 10 s after it launched. The height h , in feet, of the rocket relative to the ground at time t seconds can be modeled by the function shown.



- time
- height
- gravity

A. How are the launch and landing times related to the modeling function?

- rocket travel time

B. What additional information about the rocket launch could you use to construct an accurate model for the rocket's height relative to the ground?

- angle of the rocket
→ x & y components

C. **Construct Arguments** Charlie believes that the function $h(t) = -16t^2 + 160t + 0$ models the height of the rocket with respect to time. Do you agree? Explain your reasoning and indicate the domain of this function. © MP.3

$$\begin{aligned} h(10) &= -16(10)^2 + 160(10) \\ &= -16(100) + 1600 \\ h(10) &= 0 \end{aligned}$$

HABITS OF MIND

Reason In Charlie's function, what is the value of c ? Why is this the correct value? © MP.2

zero

Notes ✓ already factored!

ex 1) ... $f(x) = x(x-4)(x+3) \rightarrow x^3 \dots$

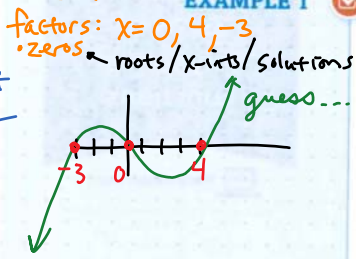
EXAMPLE 1 Try It! Use Zeros to Graph a Polynomial Function

1. Factor each function. Then use the zeros to sketch its graph.

a. $f(x) = 4x^3 + 4x^2 - 24x$

b. $g(x) = x^4 - 81$

zero product property

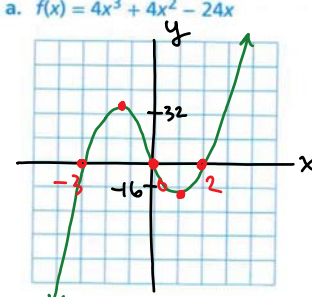


mono GCF

$4x(x^2+x-6)$

$\rightarrow 4x(x+3)(x-2)$

$x=0 \quad x=-3 \quad x=2$



diff of squares

$(x^2-9)(x^2+9)$

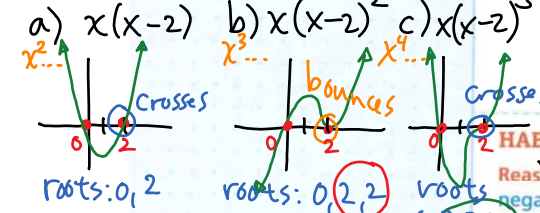
$\rightarrow (x-3)(x+3)(x^2+9)$

$x=3 \quad x=-3 \quad ?$

imag roots

Multiplicity of a zero...

What if...



EXAMPLE 2 Try It! Understand How a Multiple Zero Can Affect a Graph

2. Describe the behavior of the function at each of its zeros.

a. $f(x) = x(x+4)(x-1)^4$

b. $f(x) = (x+9)(x-1)(x+2)$

x^6

roots: 0, 4, 1, 1, 1, 1

bounces

x^9

roots: 3i, -3i, 1, 1, 1, 1, -2, -2

imag roots

Crosses bounces

HABITS OF MIND

Reason Do the values of a function always change from positive to negative or negative to positive on either side of a zero? Explain. MP2

GC

even multiplicity \rightarrow Bounces

odd multiplicity \rightarrow Crosses!

Turning Points (U-turns)

EXAMPLE 3 Try It! Find Real and Complex Zeros

3. What are all the real and complex zeros of the polynomial function shown in the graph?

$3 \mid 2 \ -8 \ 9 \ -9$

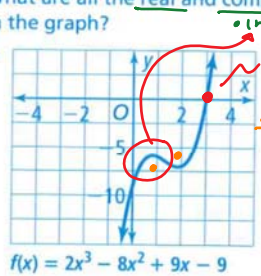
$2 \ -2 \ 3 \ 0$

$x^2 \ x \ c \ R$

$\rightarrow 3$ is a zero

$(x-3)(2x^2-2x+3)$

won't factor



$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(3)}}{2(2)}$

$= \frac{2 \pm \sqrt{4 - 24}}{4} = \frac{2 \pm \sqrt{-20}}{4} = \frac{2 \pm 2i\sqrt{5}}{4} = \frac{1 \pm i\sqrt{5}}{2}$

$\rightarrow 3, \frac{1}{2} \pm \frac{\sqrt{5}}{2}i$

$x^4?$

two real roots

got root? 2?

$f(x) = x^4 - 3x^2 - 4$

quad

$(x-2)(x+2)(x^2+1)$

$x^2+1=0$

$x^2=-1$

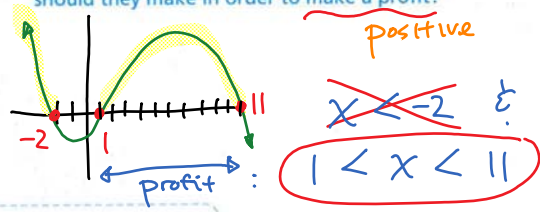
$x=\pm\sqrt{-1}$

$x=\pm i$

or quad formula

EXAMPLE 4 Try It! Interpret the Key Features of a Graph in Context

4. Due to a decrease in the cost of materials, the profit function for Acme Innovations has changed to $Q(x) = -x^3 + 10x^2 + 13x - 22$. How many lamps should they make in order to make a profit?



HABITS OF MIND

Make Sense and Persevere On a graph, how do complex roots differ from real roots? © MP.1

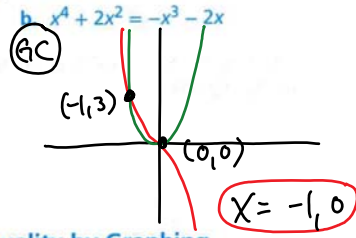
Turning point portion of graph does NOT cross x-axis....

EXAMPLE 5 Try It! Solve Polynomial Equations

5. What is the solution of the equation?

a. $x^3 - 7x + 6 = x^3 + 5x^2 - 2x - 24$

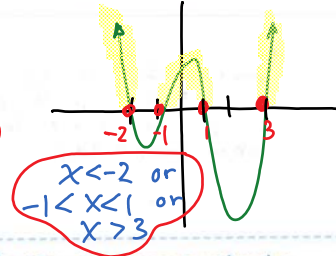
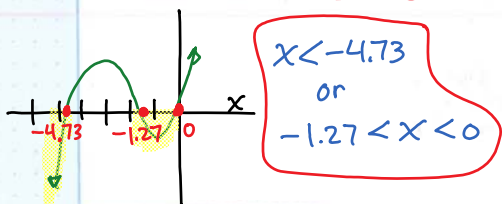
GC
or algebra
 $x^3 - 7x + 6 = 0$
 $-x^3 - 5x^2 + 2x + 24$
 $(-5x^2 - 5x + 30 = 0) \div -5$
 $x^2 + x - 6 = 0$
 $(x + 3)(x - 2) = 0$
 $x = -3, 2$



EXAMPLE 6 Try It! Solve a Polynomial Inequality by Graphing

6. What are the solutions of the inequality?

a. $2x^3 + 12x^2 + 12x < 0$ Negative b. $(x^2 - 1)(x^2 - x - 6) > 0$ positive



HABITS OF MIND

Use Structure How does solving $2x^3 + 12x^2 + 12x = 0$ help you to solve the inequality $2x^3 + 12x^2 + 12x < 0$? © MP.7

roots: pos or neg...

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are the zeros of a polynomial function related to a function's equation and graph?

2. **Error Analysis** In order to identify the zeros of the function, a student factored the cubic function $f(x) = x^3 - 3x^2 - 10x$ as follows:

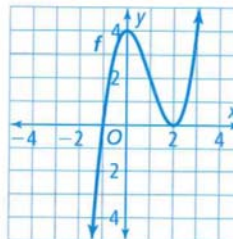
$$\begin{aligned} f(x) &= x^3 - 3x^2 - 10x \\ &= x(x^2 - 3x - 10) \\ &= x(x - 5)(x + 2) \\ x &= 0, x = -5, x = 2 \end{aligned}$$

Describe and correct the error the student made. © MP.3

3. **Make Sense and Persevere** Explain how you can determine that the function $f(x) = x^3 + 3x^2 + 4x + 2$ has both real and complex zeros. © MP.1

Do You KNOW HOW?

4. If the graph of the function f has a multiple zero at $x = 2$, what is a possible exponent of the factor $x - 2$? Justify your reasoning.



5. Energy Solutions manufactures LED light bulbs. The profit p , in thousands of dollars earned, is a function of the number of bulbs sold, x , in ten thousands. Profit is modeled by the function $-x^3 + 9x^2 - 11x - 21$. For what number of bulbs manufactured is the company profitable?