

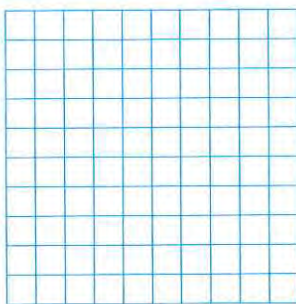
3-6

Theorems
about Roots
of Polynomial
Equations **CRITIQUE & EXPLAIN**

Look at the polynomial functions shown.

$$g(x) = x^2 - 7x - 18$$

$$h(x) = 5x^2 + 24x + 16$$



- A. Avery has a conjecture that the zeros of a polynomial function have to be positive or negative factors of its constant term. Factor g completely. Are the zeros of g factors of -18 ?

- B. **Look for Relationships** Now test Avery's conjecture by factoring $h(x)$. Does Avery's conjecture hold? If so, explain why. If not, make a new conjecture.

HABITS OF MIND

Use Structure For the factored function $k(x) = (ax + b)(cx + d)$, what are the coefficients in the expanded expression? © MP.7

**EXAMPLE 1** **Try It! Identify Possible Rational Solutions**

1. List all the possible rational solutions for each equation.

a. $4x^4 + 13x^3 - 124x^2 + 212x - 8 = 0$

b. $7x^4 + 13x^3 - 124x^2 + 212x - 45 = 0$

EXAMPLE 2 **Try It! Use the Rational Root Theorem**

2. A jewelry box measures $2x + 1$ in. long, $2x - 6$ in. wide, and x in. tall. The volume of the box is given by the function $v(x) = 4x^3 - 10x^2 - 6x$. What is the height of the box, in inches, if its volume is 28 in.^3 ?

HABITS OF MIND

Critique Arguments For the jewelry box, a student thought that the rational roots could be $\pm 6, \pm 3, \pm 2, \pm 1, \pm \frac{3}{2}, \pm \frac{1}{2}, \pm \frac{1}{4}$, using factors of -6 for the numerator and factors of 4 for the denominator of the possible rational roots. Is the student correct? Explain. © MP.3

EXAMPLE 3 **Try It! Find All Complex Roots**

3. What are all complex roots of the equation $x^3 - 2x^2 + 5x - 10 = 0$?



**EXAMPLE 4** **Try It! Irrational Roots and the Coefficients of a Polynomial**

4. Suppose a quadratic polynomial function f has two complex zeros that are a conjugate pair, $a - bi$ and $a + bi$ (where a and b are real numbers). Are all the coefficients of f real? Explain.

HABITS OF MIND

Construct Arguments Could a polynomial equation with rational coefficients have two complex roots that are not conjugates as its only roots? Explain. © MP.3

EXAMPLE 5 **Try It! Write Polynomial Functions Using Conjugates**

5. a. What is a quadratic equation in standard form with rational coefficients that has a root of $5 + 4i$?
- b. What is a polynomial function Q of degree 4 with rational coefficients such that $Q(x) = 0$ has roots $2 - \sqrt{3}$ and $5i$?

HABITS OF MIND

Reason Is it possible to write a polynomial function of degree 3 that has rational coefficients and zeros $2 - \sqrt{3}$ and $5i$? Explain. © MP.2

Do You UNDERSTAND?

ESSENTIAL QUESTION How are the roots of a polynomial equation related to the coefficients and degree of the polynomial?

- Error Analysis** Renaldo said that a polynomial equation with real coefficients has zeros $-1 + 2i$ and $3 + \sqrt{5}$ and has a degree of 4. Is Renaldo correct? Explain. © MP.3
- Reason** A fifth degree polynomial $P(x)$ with rational coefficients has zeros $2i$ and $\sqrt{7}$. What other zeros does $P(x)$ have? Explain. © MP.2
- Make Sense and Persevere** If one root of a polynomial equation is $4 + 2i$, is it certain that $4 - 2i$ is also a root of the equation? Explain. © MP.1

Do You KNOW HOW?

List all the possible rational solutions for each equation according to the Rational Roots Theorem. Then find all of the rational roots.

4. $0 = x^3 + 4x^2 - 9x - 36$

5. $0 = x^4 - 2x^3 - 7x^2 + 8x + 12$

6. $0 = 4x^3 + 8x^2 - x - 2$

7. $0 = 9x^4 - 40x^2 + 16$

A polynomial equation with rational coefficients has the given roots. List two more roots of each equation.

8. $1 + \sqrt{11}$ and $-3 + \sqrt{17}$

9. $5 + 12i$ and $-9 - 7i$

10. $12 + 5i$ and $6 - \sqrt{13}$

11. $5 - 15i$ and $17 + \sqrt{23}$