

3-7

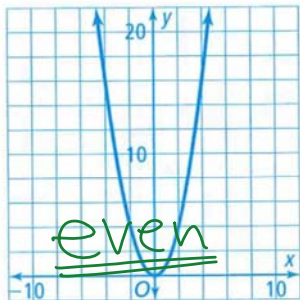
Transformations of Polynomial Functions

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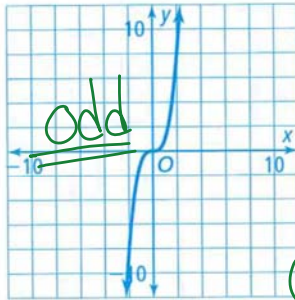
EXPLORE & REASON

Look at the polynomial graphs below.

$$f(x) = x^2$$



$$g(x) = x^3$$



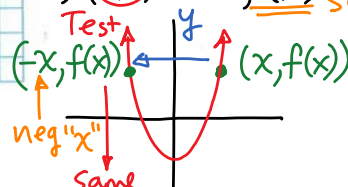
A. Is the graph of f or g symmetric about the y -axis? Is the graph of f or g symmetric about the origin? Explain.



horiz reflection

Even Function

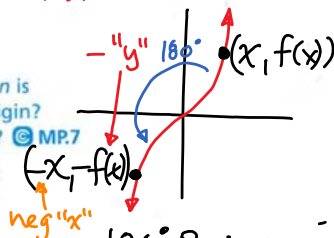
$$f(-x) \rightarrow f(x) \text{ Same}$$



• Symmetry about the y -axis

Odd Function

$$f(-x) \rightarrow -f(x) \text{ opposite}$$

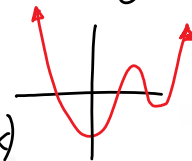


• 180° Rotational Symmetry

Neither

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$



B. **Look for Relationships** Graph more functions of the form $y = x^n$ where n is a natural number. Which of these functions are symmetric about the origin? Which are symmetric about the y -axis? What conjectures can you make? © MP.7

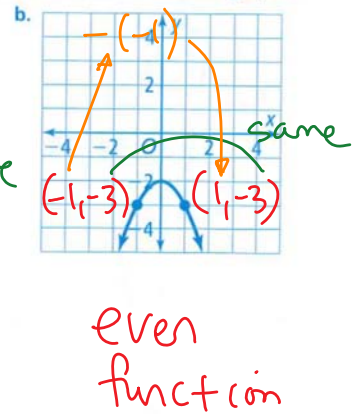
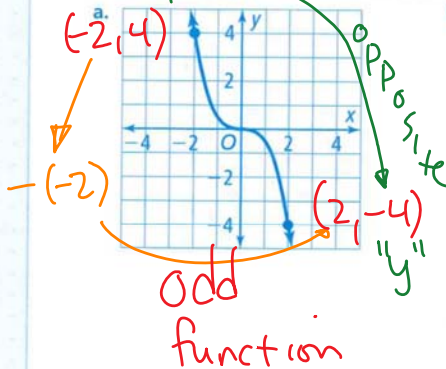


HABITS OF MIND

Look for Relationships Do you notice any other patterns among the functions with even degree or the functions with odd degree? © MP.7

EXAMPLE 1 Try It! Identify Even and Odd Functions From Their Graphs

1. Classify the polynomial functions as even or odd based on the graphs.



EXAMPLE 2 Try It! Identify Even and Odd Functions From Their Equations

2. Is the function odd, even, or neither?

Test:
 $f(-x)$
 $\rightarrow f(x)$: even
 $\rightarrow -f(x)$: odd

a. $f(x) = 7x^5 - 2x^2 + 4$

$$f(-x) = 7(-x)^5 - 2(-x)^2 + 4$$

$$= -7x^5 - 2x^2 + 4$$

not same: **not EVEN**

... opposite?

$$-(7x^5 - 2x^2 + 4)$$

$$\rightarrow -7x^5 + 2x^2 - 4$$

not opposite: **not ODD**

neither

b. $f(x) = x^6 - 2$

$$f(-x) = (-x)^6 - 2$$

$$= x^6 - 2$$

same: **EVEN**

HABITS OF MIND

Make Sense and Persevere Why do you replace x with $-x$ when determining if a function is odd, even, or neither? © MP.1

EXAMPLE 3 Try It! Graph Transformations of Cubic and Quartic Parent Functions

3. How does the graph of the function $g(x) = 2x^3 - 5$ differ from the graph of its parent function?

$$y = a(x-h)^n + k$$

ex 3b) $h(x) = 5(x-2)^3 + 4$

vert stretch by 5

$h: 2$ right 2

$k: 4$ up 4

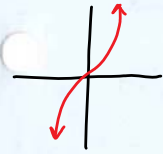
x^3 x^4

$a: 2$ vert stretch

$k: -5$ down 5

EXAMPLE 4 Try It! Identify a Transformation

4. Determine the equation of each graph as it relates to its parent cubic function or quartic function.



$$y = a(x-h)^3 + k$$

$$y = a(x-2)^3 + 4$$

Stretch?

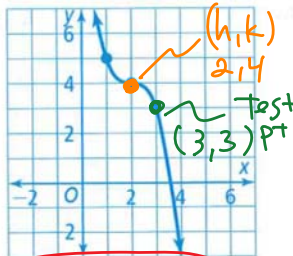
$$3 = a(3-2)^3 + 4$$

$$3 = a(1)^3 + 4$$

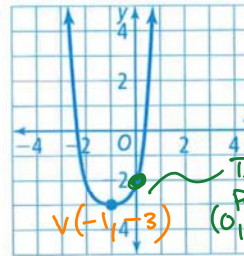
$$3 = a + 4$$

$$-1 = a$$

$$\therefore y = -1(x-2)^3 + 4$$



b.



$$y = a(x-h)^4 + k$$

$$y = a(x-(-1))^4 - 3$$

$$-2 = a(0+1)^4 - 3$$

$$-2 = a(1) - 3$$

$$-2 = a - 3$$

$$1 = a$$

$$\therefore y = 1(x+1)^4 - 3$$

HABITS OF MIND

Look for Relationships What type of transformation would change a function's end behavior? © MP.7

$$a < 0$$

negative

EXAMPLE 5 Try It! Apply a Transformation of a Cubic Function

5. a. The volume of a cube, in cubic feet, is given by the function $V(x) = x^3$. Write a function for the volume of the cube with cubic inches as the units.

b. A storage unit is in the shape of a rectangular prism. The volume of the storage unit is given by $V(x) = (x)(x)(x-1) = x^3 - x^2$. A potential customer wants to compare the volume of this storage unit with that of another storage unit that is 1 foot longer in every dimension. Write a function for the volume of this larger unit.

HABITS OF MIND

Critique Arguments A student thought that, for 5a, the new function should be $V(x) = 144x^3$. What are the two errors the student made? © MP.3

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are symmetry and transformations represented in the graph and equation of a polynomial function?

2. **Vocabulary** What is the difference between the graph of an even function and the graph of an odd function?

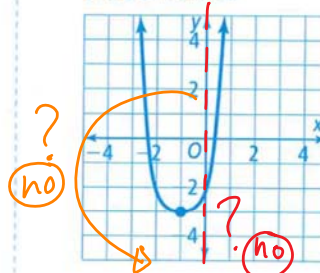
3. **Error Analysis** A student identified the transformations of the polynomial function $f(x) = 3(x - 1)^3 - 6$ as follows:

The function is shifted to the left 1 unit, stretched vertically, and is shifted downward 6 units.

Describe and correct the error the student made. © MP.6

Do You KNOW HOW?

4. Classify the function on the graph as odd, even, or neither.



→ neither

5. Use the equation to classify the function as odd, even, or neither.

$$g(x) = 4x^3 - x$$

$$g(-x) = 4(-x)^3 - (-x) = -4x^3 + x$$

Opposite → ODD

not same: not EVEN

6. The volume of cardboard box is given by the function $V(x) = x(x - 2)(x) = x^3 - 2x^2$. Write a new function for the volume of a cardboard box that is 2 units longer in every dimension.