

4-1

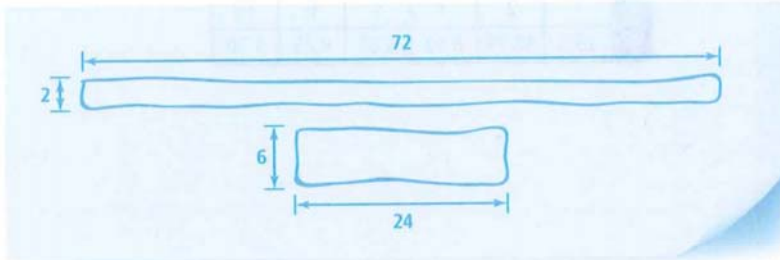
Inverse Variation and the Reciprocal Function

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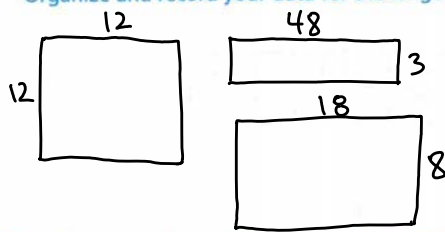
MODEL & DISCUSS

$$A = l \cdot w$$

The two rectangles shown both have an area of 144 square units.



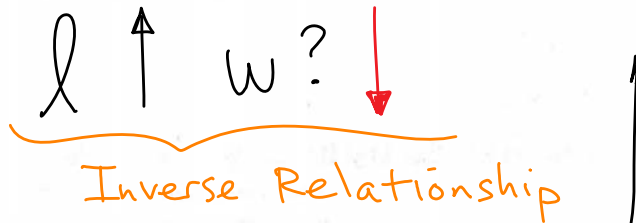
- A. Sketch as many other rectangles as you can that have the same area. Organize and record your data for the lengths and widths of the rectangles.



$$A = 144$$

l	w
1	144
2	72
3	48
4	36
6	24
8	18
9	16
12	12

- B. Use Structure Considering rectangles with an area of 144 square units, what happens to the width of the rectangle as the length increases? © MP.7



- C. Examine at least five other pairs of rectangles, each pair sharing the same area. How would you describe the relationship between the lengths and widths?

HABITS OF MIND

Use Structure How does the formula for the area of a rectangle make sense with the relationships you found? © MP.7

$$\frac{A}{l} = \frac{lw}{l} \Rightarrow \frac{A}{l} = w$$

Inverse

rational equation

$$y = \frac{k}{x}$$

Constant of variation
• not of (h, k)

Notes

Assess

EXAMPLE 1 Try It! Identify Inverse Variation

1. Determine if each table of values represents an inverse variation.

A.

x	1	2	3	4	6	12
y	25.5	12.75	8.50	5.10	4.25	1.70

• $xy \Rightarrow 12$ **Yes**

B.

x	6.6	5.5	4.4	3.3	2.2	1.1
y	3	5	7	9	11	13

20 17 14 11 8 5

• $xy \Rightarrow$ not the same product **No**

$$y = \frac{k}{x}$$

$$xy = k$$

EXAMPLE 2 Try It! Use Inverse Variation

Find k.

2. In an inverse variation, $x = 6$ and $y = \frac{1}{2}$

a. What is the equation that represents the inverse variation?

$$y = \frac{k}{x} \rightarrow \frac{1}{2} = \frac{k}{6} \rightarrow 1(6) = 2k$$

$$3 = k \therefore y = \frac{3}{x}$$

b. What is the value of y when $x = 15$?

$$y = \frac{3}{15} \rightarrow y = \frac{1}{5}$$

Note: Proportion $\frac{a}{b} = \frac{c}{d} \leftrightarrow ad = bc$ *Cross mult*

HABITS OF MIND

Construct Arguments For rectangles that have a constant perimeter, the length increases as the width decreases. Is the relationship between the length and width an inverse variation? Explain. **MP3**

distance around

$$P = 2l + 2w$$

EXAMPLE 3 Try It! Use an Inverse Variation Model

3. The amount of time it takes for an ice cube to melt varies inversely to the air temperature, in degrees. At 70°F, the ice will melt in 20 min. How long will it take the ice to melt if the temperature is 85°F?

$y = \frac{k}{x \text{ time}}$

Find k.

Proportion
* swap diagonal

ex) $\frac{1}{2} = \frac{10}{20}$

$\frac{20}{2} = \frac{10}{1}$

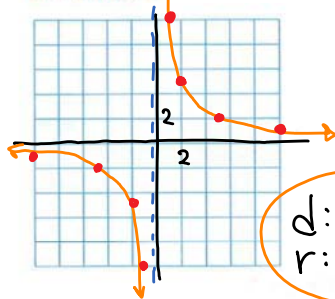
$70 = \frac{k}{20} \dots \underline{1400} = k$

$y = \frac{1400}{x} \rightarrow \frac{85 = 1400}{x}$

$\rightarrow \frac{x}{1} = \frac{1400}{85} \rightarrow x = 16.5 \text{ min}$

EXAMPLE 4 Try It! Graph the Reciprocal Function

4. Graph the function $y = \frac{10}{x}$. What are the domain, range, and asymptotes of the function?



x	y
1	10
2	5
0	undef

d: $x \neq 0$
r: $y \neq 0$

vert asympt: $x=0$

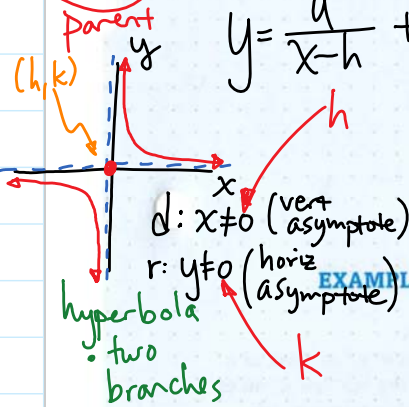
horiz asympt: $y=0$

Reciprocal Function

$y = \frac{1}{x}$

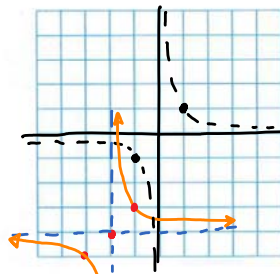
• transformation

$y = \frac{a}{x-h} + k$



EXAMPLE 5 Try It! Graph Translations of the Reciprocal Function

5. Graph $g(x) = \frac{1}{x+2} - 4$. What are the equations of the asymptotes? What are the domain and range?



a: 1
h: -2
k: -4

d: $x \neq -2$
r: $y \neq -4$

vert asympt: $x=2$

horiz asympt: $y=4$

★ $y = \frac{1}{3x-6} + 5$

Hmm...

a? $y = \frac{a}{x-h} + k$
h?
k?

$\frac{1}{3(x-2)} + 5$
h: 2 k: 5

HABITS OF MIND

Communicate Precisely A translation of the reciprocal function has a vertical asymptote at $x=5$. What is a possible equation for the function? © MP.6

$y = \frac{a}{x-h} + k$ $x=h$
 $y = \frac{a}{x-5} + k \rightarrow y = \frac{1}{x-5}$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are inverse variation and reciprocal functions related and represented?

2. **Construct Arguments** Explain why the amount of propane in a grill's tank and the time spent grilling represents an inverse variation. © MP.3



3. **Vocabulary** Why is it impossible for the graph of the function $y = \frac{1}{x}$ to intersect the horizontal asymptote at the x -axis?

4. **Error Analysis** Carmen said the table of values shown represents an inverse variation. Explain why Carmen is mistaken. © MP.3

x	1	2	3	4	8	16
y	24	12	8	6	3	2

Do You KNOW HOW?

5. In an inverse variation, $x = -8$ when $y = -\frac{1}{4}$. What is the value of y when $x = 4$?

$y = \frac{k}{x}$ Constant of Variation

$$-\frac{1}{4} = \frac{k}{-8} \Rightarrow -1(-8) = 4k$$

$$2 = k$$

$$y = \frac{2}{x} \Rightarrow y = \frac{2}{4} = \boxed{\frac{1}{2}}$$

6. What are the equations of the asymptotes of the function $f(x) = \frac{1}{x-5} + 3$? What are the domain and range?

$$y = \frac{a}{x-h} + k \quad \begin{array}{l} h: 5 \rightarrow x=5 \text{ (vert asympt)} \\ k: 3 \rightarrow y=3 \text{ (horiz asympt)} \end{array}$$

$d: x \neq 5$
 $r: y \neq 3$

7. Suppose the amount of gas in a fuel tank of a truck varies inversely with the number of miles traveled. Model a relationship between the amount of gas in a fuel tank of a truck and the number of miles traveled by the truck as an inverse variation.

