

EXPLORE & REASON

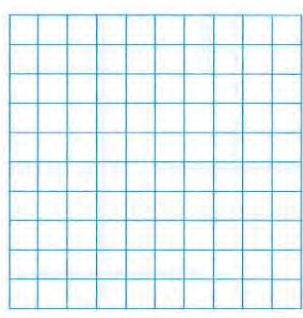
Look at the three functions shown.

$$f(x) = x - 1$$

$$g(x) = \frac{x - 1}{2}$$

$$h(x) = \frac{x - 1}{x - 2}$$

A. **Look for Relationships** Graph each function. Determine which of the functions are linear. Find the y-intercept of each function and the slope, if appropriate. © MP.7



B. What is the effect on the graph of  $f$  when dividing  $x - 1$  by 2?

C. What happens to the graph of  $h$  as  $x$  approaches 2?

D. **Communicate Precisely** What is the effect on the graph of  $f(x)$  when dividing  $x - 1$  by  $x - 2$ ? (Hint: Compare it to what you found in part (b).) © MP.6

HABITS OF MIND

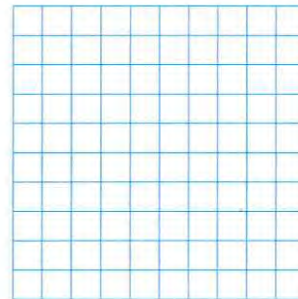
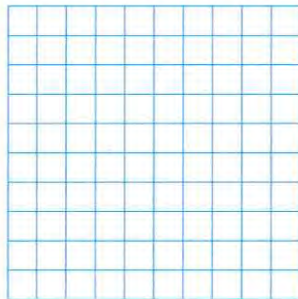
**Look for Relationships** What similarities do you notice between the graph of  $h(x) = \frac{x - 1}{x - 2}$  and the graph of a reciprocal function? © MP.7

**EXAMPLE 1** **Try It! Rewrite a Rational Function to Identify Asymptotes**

1. Use long division to rewrite each rational function. Find the asymptotes of  $f$  and sketch the graph.

a.  $f(x) = \frac{6x}{2x + 1}$

b.  $\frac{x}{x - 6}$

**EXAMPLE 2** **Try It! Find Asymptotes of a Rational Function**

2. What are the vertical and horizontal asymptotes of the graph of each function?

a.  $g(x) = \frac{2x^2 + x - 9}{x^2 - 2x - 8}$

b.  $f(x) = \frac{3x - 2}{x^2 + 7x + 12}$

**HABITS OF MIND**

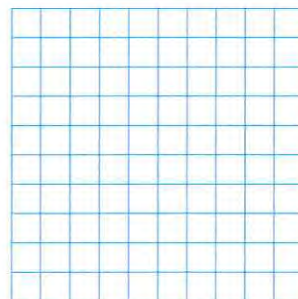
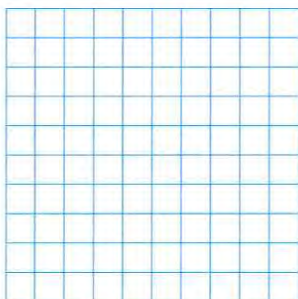
**Model With Mathematics** Under what conditions could there be a horizontal asymptote at  $y = -2$ ? Give an example. © MP.4

**EXAMPLE 3** **Try It! Graph a Function of the Form  $\frac{ax + b}{cx + d}$** 

3. Graph each function.

a.  $f(x) = \frac{4x - 3}{x + 8}$

b.  $\frac{3x + 2}{x - 1}$



**EXAMPLE 4** **Try It! Use a Rational Function Model**

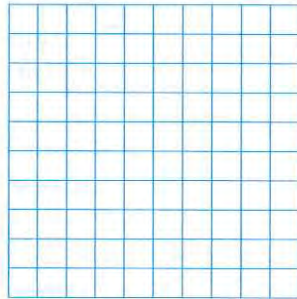
4. New techniques have changed the cost-benefit function. For the new function  $g(p) = \frac{3.2p + 1}{100 - p}$ , what percent of the pollutant can be removed for \$50 million?

**HABITS OF MIND**

**Make Sense and Persevere** What are the asymptotes for the function  $g(p) = \frac{3.2p + 1}{100 - p}$ ? © MP.1

**EXAMPLE 5** **Try It! Graph a Rational Function**

5. Identify the asymptotes and sketch the graph of  $g(x) = \frac{x^2 - 5x + 6}{2x^2 - 10}$ .

**HABITS OF MIND**

**Reason** When will the graph of a rational function have two vertical asymptotes? © MP.2

**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How can you graph a rational function?

2. **Vocabulary** Why does it make sense to call the expressions in this lesson *rational* functions?

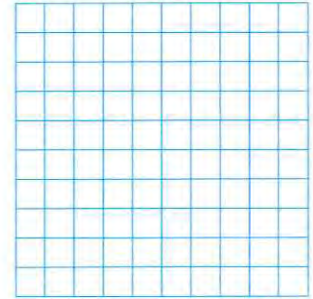
3. **Error Analysis** Ashton said the graph of  $f(x) = \frac{x+2}{2x^2+4x-6}$  has a horizontal asymptote at  $y = \frac{1}{2}$ . Describe and correct Ashton's error. © MP.3

4. **Reason** When will the graph of a rational function have no vertical asymptotes? Give an example of such a function. © MP.2

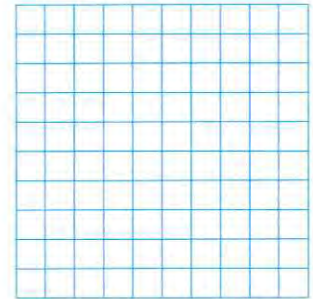
**Do You KNOW HOW?**

Find the vertical asymptote(s) and horizontal asymptote(s) of the rational function. Then graph the function.

5.  $f(x) = \frac{x+2}{x-3}$



6.  $f(x) = \frac{x-1}{2x+1}$



7. A trainer mixed water with an electrolyte solution. The relationship can be modeled by  $f(x) = \frac{4}{x+12}$ . Graph the function.

