Activity

4-2 Graphing Rational Functions

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EXPLORE & REASON

Look at the three functions shown.

$f(x) = x - 1$

$g(x) = \frac{x-1}{2}$ → $\frac{1}{2}(x-1)$

$h(x) = \frac{x-1}{x-2}$

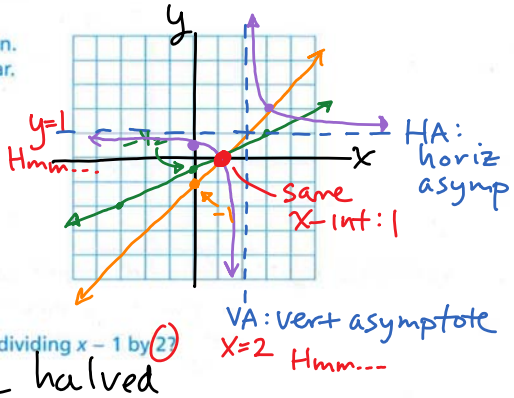
A. Look for Relationships Graph each function. Determine which of the functions are linear. Find the *y*-intercept of each function and the slope, if appropriate. MP.7

set $x=0$:

$f(0) = 0 - 1 = -1$

$g(0) = \frac{0-1}{2} = -\frac{1}{2}$

$h(0) = \frac{0-1}{0-2} = \frac{1}{2}$



B. What is the effect on the graph of *f* when dividing $x - 1$ by 2?

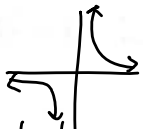
- y values are halved

C. What happens to the graph of *h* as *x* approaches 2?

→ interactions w/asymptotes

D. Communicate Precisely What is the effect on the graph of $f(x)$ when dividing $x - 1$ by $x - 2$? (Hint: Compare it to what you found in part (b).) MP.6

→ no longer a line...



- hyperbola

HABITS OF MIND

Look for Relationships What similarities do you notice between the graph of $h(x) = \frac{x-1}{x-2}$ and the graph of a reciprocal function? MP.7

$\frac{1}{x}$

- two branches
- two asymptotes

$\frac{a}{x-h}+k$ simplify $\frac{P(x)}{Q(x)}$ rational expression ex) $\frac{1}{3}+7 \to \frac{1}{3}+\frac{7\cdot3}{1\cdot3} \to \frac{1}{3}+\frac{21}{3} \to \frac{22}{3}$

long ÷ Polynomials

$3\overline{)22}$ (−21, R1) $\to 7+\frac{1}{3}$ or $7\frac{1}{3}$

Notes Assess

quotient

EXAMPLE 1 **Try It! Rewrite a Rational Function to Identify Asymptotes**

1. Use long division to rewrite each rational function. Find the asymptotes of f and sketch the graph.

a. $f(x)=\frac{6x}{2x+1}$

$2x+1\overline{)6x+0}$ (−) $6x+3$, -3 ← remainder

$\to 3+\frac{-3}{2x+1}$ or $\frac{-3}{2x+1}+3$

$a=\frac{-3}{2(x+\frac{1}{2})}$, $h:-\frac{1}{2}$, $k:3$

HA: $y=3$; VA: $x=-\frac{1}{2}$

x	y
0	0
−1	6

b. $\frac{4x}{x-3}$

$x-3\overline{)4x+0}$ (−) $4x-12$, R 12

$4+\frac{12}{x-3}$

$a=12$, $h:3$, $k:4$

HA: $y=4$; VA: $x=3$

x	y
2	−8
1	−2
0	0

Rational Expression $\frac{P(x)}{Q(x)}$ polynomials, $Q(x)\neq0$

vertical asymptotes (not part of domain)

a & c: Leading Coefficients of $P(x)$ & $Q(x)$

horizontal asymptotes (Long ÷) (not part of range?)

Case ① If degree of $P(x)$ < degree of $Q(x)$: Then $y=0$

Case ② If degree of $P(x)$ = degree of $Q(x)$: Then $y=\frac{a}{c}$

Case ③ If degree of $P(x)$ > degree of $Q(x)$: Then there are no HA.

EXAMPLE 2 **Try It! Find Asymptotes of a Rational Function**

2. What are the vertical and horizontal asymptotes of the graph of each function?

a. $g(x)=\frac{2x^2+x-9}{x^2-2x-8}$ Case ② #2

VA: $x^2-2x-8=0$, $(x-4)(x+2)=0$, $x-4=0$ ($x=4$), $x+2=0$ ($x=-2$)

HA: $y=\frac{a}{c}$, $y=\frac{2}{1}$

b. $f(x)=\frac{3x-2}{x^2+7x+12}$ Case ① #1

VA: $(x+3)(x+4)=0$, $x=-3$, $x=-4$

HA: $y=0$

HABITS OF MIND

Model With Mathematics Under what conditions could there be a horizontal asymptote at $y=-2$? Give an example. MP.4

$\frac{-2x^1+\dots}{x^1+\dots}$ same powers: Case #2

EXAMPLE 3 **Try It! Graph a Function of the Form $\frac{ax+b}{cx+d}$**

3. Graph each function.

a. $f(x)=\frac{4x-3}{x+8}$

x-intercepts: set $y=0$; $f(x)=\frac{P(x)}{Q(x)}$; $0=\frac{P(x)}{Q(x)}$; $0=P(x)$ ← 0 = numerator of rational

x-int: $4x-3=0$, $x=\frac{3}{4}$

y-ints: set $x=0$

VA: $x+8=0$, $x=-8$

HA: $\deg P(x)=\deg Q(x)$, Case ② $y=\frac{4}{1}$

y-int: $f(0)=\frac{4(0)-3}{(0)+8}$, $y=-\frac{3}{8}$

x	y
−9	39

b. $\frac{3x+2}{x-1}$

x-int: $3x+2=0$, $x=-\frac{2}{3}$

y-int: $y=\frac{3(0)+2}{0-1}=\frac{-2}{1}$

VA: $x-1=0$, $x=1$

HA: $y=\frac{3}{1}$

x	y
2	$\frac{3(2)+2}{2-1}=\frac{8}{1}$

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Notes

Assess

EXAMPLE 4 **Try It! Use a Rational Function Model**

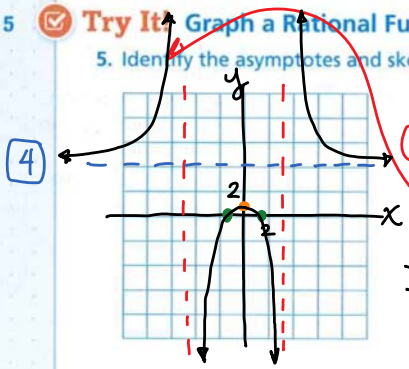
4. New techniques have changed the cost-benefit function. For the new function $g(p) = \frac{3.2p + 1}{100 - p}$, what percent of the pollutant can be removed for $50 million?

HABITS OF MIND

Make Sense and Persevere What are the asymptotes for the function $g(p) = \frac{3.2p + 1}{100 - p}$? MP.1

EXAMPLE 5 **Try It! Graph a Rational Function**

5. Identify the asymptotes and sketch the graph of $g(x) = \frac{x^2 - 5x + 6}{2x^2 - 18}$.



diff of squares $(2x-3)(2x+3)$

$\frac{4x^2-9}{x^2+2x-15}$

$(x+5)(x-3)$

VA

$(x+5)(x-3)=0$

$x=-5 \mid x=3$

HA

deg P(x) = deg Q(x):

$y = \frac{4}{1}$

X-int

$(2x-3)(2x+3)=0$

$x=\frac{3}{2} \mid x=-\frac{3}{2}$

x	y
−6	$\frac{4(-6)^2-9}{(-6)^2+2(-6)-15}$ $\frac{144-9}{36-12-15} = \frac{135}{9}$ Big
4	... Big

y-int

$y = \frac{4(0)^2-9}{0^2+2(0)-15} = \frac{9}{15} = \frac{3}{5}$

HABITS OF MIND

Reason When will the graph of a rational function have two vertical asymptotes? MP.2

Q(x)'s degree = 2

denominator

Assess

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you graph a rational function?

2. **Vocabulary** Why does it makes sense to call the expressions in this lesson *rational* functions?

3. **Error Analysis** Ashton said the graph of $f(x) = \frac{x+2}{2x^2+4x-6}$ has a horizontal asymptote at $y = \frac{1}{2}$. Describe and correct Ashton's error. MP.3

4. **Reason** When will the graph of a rational function have no vertical asymptotes? Give an example of such a function. MP.2

Do You KNOW HOW?

Find the vertical asymptote(s) and horizontal asymptote(s) of the rational function. Then graph the function.

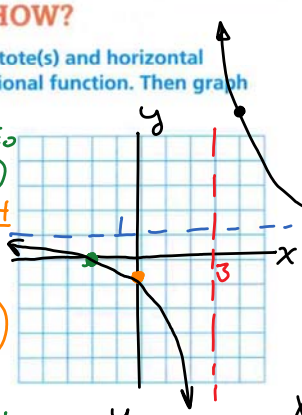
5. $f(x) = \frac{x+2}{x-3}$

VA: $x-3=0$, $x=3$

HA: degs =, $y=\frac{a}{c}=\frac{1}{1}$

x-int: $x+2=0$, $x=-2$

y-int: $\frac{0+2}{0-3}=-\frac{2}{3}$



x	y
4	$\frac{4+2}{4-3}=6$

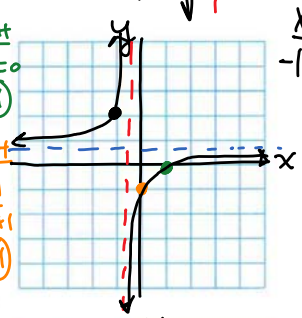
6. $f(x) = \frac{x-1}{2x+1}$

VA: $2x+1=0$, $x=-\frac{1}{2}$

HA: degs =, $y=\frac{a}{c}=\frac{1}{2}$

x-int: $x-1=0$, $x=1$

y-int: $\frac{0-1}{2(0)+1}=-1$



x	y
-1	$\frac{-1-1}{2(-1)+1}=\frac{-2}{-1}=2$

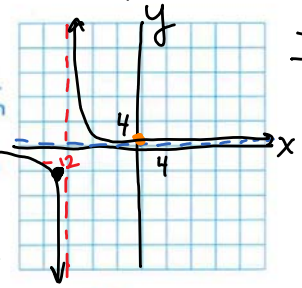
7. A trainer mixed water with an electrolyte solution. The relationship can be modeled by $f(x) = \frac{4}{x+12}$. Graph the function.

VA: $x+12=0$, $x=-12$

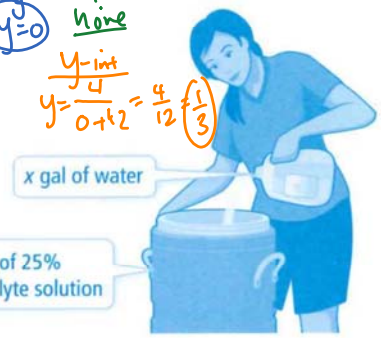
HA: degs <, $y=0$

x-int: $4=0$, ? none

y-int: $y=\frac{4}{0+12}=\frac{4}{12}=\frac{1}{3}$



x	y
-13	$-13+12=-4$



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