

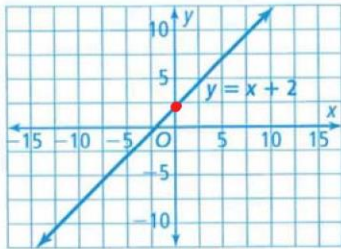
Factor Polynomials: mono, diff of sum; diff of cubes, GCF, squares, factor by grouping  
 mono/bi/tri "family of terms"

$\times$   $\div$

**4-3**  
 Multiplying and Dividing Rational Expressions  
 PearsonRealize.com

**EXPLORE & REASON**

Consider the following graph of the function  $y = x + 2$ .



$m=1$   $b=2$

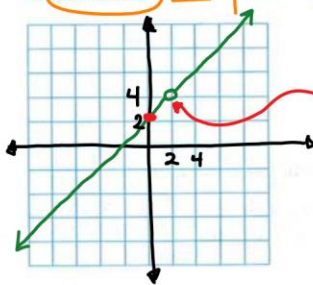
A. What is the domain of this function?

$d: \mathbb{R}$

$x$	$y$
all real #s	$\mathbb{R}$

\* Hyperbolas:  
 asymptotes  
 • not part of the domain...

B. Sketch a function that resembles the graph, but restrict its domain to exclude 2. *not part of domain*



$y = x + 2$   
 but  $x \neq 2$   
 Hole?

C. Look for Relationships Consider the function you have sketched. What kind of function might have a graph like this? Explain. © MP.7

$$\frac{P(x)}{Q(x)} = \frac{(x+2)(x-2)}{(x-2)}$$

$x \neq 2$   
 $x-2 \neq 0$

$$\frac{P(x)}{Q(x)} = \frac{x^2 - 4}{x - 2}$$

Vert asymptote  
 → Hole: vert asymptote factor CANCELS...

**HABITS OF MIND**

Reason Does the graph of  $y = \frac{2x+6}{x+3}$  have a vertical asymptote at  $x = -3$ ? Explain. © MP.2

$y = \frac{(2x+6)}{(x+3)} \xrightarrow{\text{mono GCF}} \frac{2(x+3)}{(x+3)}$

binomials  
 Hole at  $x = -3$

**EXAMPLE 1** Try It! Write Equivalent Rational Expressions

mono GCF

1. Write an expression equivalent to  $\frac{3x^5 - 18x^4 - 21x^3}{2x^6 - 98x^4}$ .  
Remember to give the domain for your expression.

$$\frac{3x^3(x^2 - 6x - 7)}{2x^4(x^2 - 49)} \rightarrow \frac{3x^3(x-7)(x+1)}{2x^4(x-7)(x+7)}$$

diff of squares

$X=0$  Hole |  $X=7$  Hole |  $X=-7$  ← Vert asymptotes?

$d: x \neq 0, \pm 7$

$$\frac{3(x+1)}{2x(x+7)} \text{ or } \frac{3x+3}{2x^2+14x}$$

**EXAMPLE 2** Try It! Simplify a Rational Expression

2. Simplify each expression and show the domain for which the identity with the two expressions is valid.

a.  $\frac{x^2+2x+1}{x^3-2x^2-3x} \rightarrow x(x^2-2x-3)$

b.  $\frac{x^3+4x^2-x-4}{x^2+3x-4}$  factor by grouping

$x^2 \begin{matrix} x^3 & 4x^2 \\ -x & -4 \end{matrix}$  # terms: even GCF rows & columns

$$\frac{(x+1)(x+1)}{x(x-3)(x+1)} = \frac{x+1}{x(x-3)}$$

$X=0$  VA |  $X=3$  VA |  $X=-1$  Hole

$d: x \neq 0, 3, -1$

or

$$\frac{x+1}{x^2-3x}$$

$(x-1)(x+1)(x+4)$   
 $(x+4)(x-1)$

$X=-4$  Hole |  $X=1$  Hole

$d: x \neq -4, 1$

$\rightarrow x+1$

**HABITS OF MIND**

**Critique Reasoning** Bailey simplified the rational expression  $\frac{x^2+2x+4}{x^2+x+2}$  by dividing out the  $x^2$ -terms, and then dividing out a factor of  $x+2$  to get 2 as the simplified form of the rational expression. Is Bailey correct? Why or why not?

MP3

illegal

**EXAMPLE 3** Try It! Multiply Rational Expressions

3. Find the simplified form of each product, and give the domain.

a.  $\frac{(x^2-16)(x^2+x-90)}{(9-x)(x^2+14x+40)}$

$$\frac{(x-4)(x+4)(x-9)(x+10)}{(9-x)(x+4)(x+10)}$$

hole hole hole

$d: x \neq 9, -4, -10$

$1(-9+x) \rightarrow \frac{x-4}{-1} \text{ or } -x+4$

b.  $\frac{(x+3)(3x-18)}{4x(6x+18)} \cdot \frac{x^2}{4x+12}$

$$\frac{(x+3) \cdot 3(x-6) \cdot x \cdot x}{4 \cdot x \cdot 6(x+3) \cdot 4(x+3)}$$

hole hole

$d: x \neq 0, -3$

$\rightarrow \frac{x(x-6)}{32(x+3)}$

**EXAMPLE 4** **Try It!** Multiply a Rational Expression by a Polynomial

4. Find the simplified form of each product and the domain.

a.  $\frac{(x^3-4x)}{(x^2+13x-5)} \cdot (2x^3-3x^2-5x)$       b.  $\frac{3x^2+6x}{x^2-49} \cdot (x^2+9x+14)$

*Handwritten:*  $\frac{x(x^2-4)}{(3x+5)(2x-5)} \cdot (2x^3-3x^2-5x)$        $\frac{3x(x+2)(x+7)(x+2)}{(x-7)(x+7)}$

*Handwritten domain for b:*  $d: x \neq 7, -7$

**HABITS OF MIND**

**Generalize** Why is it important to identify the domain of a rational expression before you simplify it rather than after? **MP.8**

*Handwritten:*  $\frac{x(x-2)(x+2) \cdot (2x-5)(x+1)}{(3x+1)(2x-5)}$

*Handwritten domain:*  $d: x \neq -\frac{1}{3}, \frac{5}{2}$

**÷ Fractions**

*Handwritten notes:*  
 → mult by reciprocal  
 → "skip, flip, & mult"  
 → "KFC"  
 1st part  
 2nd part  
 reciprocal  
 sign

**EXAMPLE 5** **Try It!** Divide Rational Expressions

5. Find the simplified quotient and the domain of each expression.

a.  $\frac{1}{(x^2+9x)} \div \frac{(6-x)}{(3x^2-18x)}$       b.  $\frac{2x^2-12x}{x+5} \div \frac{(x-6)}{(x+5)}$

*Handwritten:*  $\frac{1}{x(x+9)} \cdot \frac{3x(x-6)}{-1(-6+2)}$        $\frac{2x(x-6)}{(x+5)} \cdot \frac{(x+5)}{(x-6)}$

*Handwritten domain for a:*  $d: x \neq 0, -9, 6$       *Handwritten domain for b:*  $d: x = -5, 6$        $2x$

**EXAMPLE 6** **Try It!** Use Division of Rational Expressions

6. The company compares the ratios of surface area to volume for two more containers. One is a rectangular prism with a square base. The other is a rectangular prism with a rectangular base. One side of the base is equal to the side-length of the first container, and the other side is twice as long. The surface area of this second container is  $4x^2 + 6xh$ . The heights of the two containers are equal. Which has the smaller surface area-to-volume ratio?



*Handwritten:* Hmm....

**HABITS OF MIND**

**Use Structure** Is the domain of the quotient  $\frac{2x^2-12x}{x+5} \div \frac{(x-6)}{(x+5)}$  different from the domain of the product  $(\frac{2x^2-12x}{x+5})(\frac{x-6}{x+5})$ ? Explain **MP.7**

**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How does understanding operations with fractions help you multiply and divide rational expressions?

2. **Vocabulary** In your own words, define rational expression and provide an example of a rational expression.

3. **Error Analysis** A student divided the rational expressions as follows:  
 $\frac{4x}{5y} \div \frac{20x^2}{25y^2} = \frac{4x}{5y} \div \frac{4 \cdot 20}{25y^2} = \frac{16x}{25y^3}$   
 Describe and correct the errors the student made. © MP.3

4. **Make Sense and Persevere** Why state the domain when simplifying rational expressions? © MP.1

**Do You KNOW HOW?**

5. What is the simplified form of the rational expression  $\frac{x^2 - 36}{x^2 + 3x - 18}$ ? What is the domain?

$$\frac{(x-6)(x+6)}{(x+6)(x-3)} \rightarrow \frac{x-6}{x-3}$$

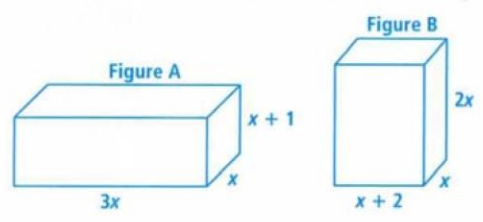
$d: x \neq -6, 3$

6. Find the product and give the domain of  $\frac{(y+3)(y^2+4y+4)}{(y+2)(y^2-9)}$ .

$$\frac{(y+3)(y+2)(y+2)}{(y+2)(y-3)(y+3)} \rightarrow \frac{y+2}{y-3}$$

$d: y \neq -2, 3, -3$

7. Find and simplify the ratio of the volume of Figure A to the volume of Figure B.



$$\frac{3x \cdot x \cdot (x+1)}{(x+2)(x)(2x)} \leftarrow \text{ratio}$$

$$\rightarrow \frac{3(x+1)}{2(x+2)}$$