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# 4-4

## Adding and Subtracting Rational Expressions

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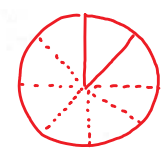
### CRITIQUE & EXPLAIN

Teo and Shannon find the following exercise in their homework:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$$

- A. Teo claims that a common denominator of the sum is  $2 + 3 + 9 = 14$ . Shannon claims that it is  $2 \cdot 3 \cdot 9 = 54$ . Is either student correct? Explain why or why not.

Shannon is correct.  
 → need same divisor...



- B. Find the sum, explaining the method you use.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$$

$\downarrow$       $\downarrow$       $\downarrow$   
 2 ✓    3 ✓    3·3 ✓

**LCD**

- Common factors
- ...then include other factors →

Common denominator  
 $2 \cdot 3 \cdot 9 \rightarrow 54$   
 • not the most efficient....  
 → Find LCD, least common denominator

$3 \cdot 2 \cdot 3 \rightarrow 18$

- C. **Construct Arguments** Timothy states that the quickest way to find the sum of any two fractions with unlike denominators is to multiply their denominators to find a common denominator, and then rewrite each fraction with that denominator. Do you agree? © MP.3

### HABITS OF MIND

**Look for Relationships** For two fractions with denominators 10 and x, when is  $10x$  the least common multiple? When is  $10x$  NOT the least common multiple? © MP.7

**EXAMPLE 1** **Try It!** Add Rational Expressions With Like Denominators

1. Find the sum.

a.  $\frac{10x-5}{2x+3} + \frac{8-4x}{2x+3}$

$$\rightarrow \frac{10x-5+8-4x}{2x+3}$$

$$\rightarrow \frac{6x+3}{2x+3}$$

*Handwritten note:  $3(2x+1)$*

b.  $\frac{x-5}{x+5} + \frac{3x-21}{x+5}$

$$\rightarrow \frac{x-5+3x-21}{x+5}$$

$$\rightarrow \frac{4x-26}{x+5}$$

*Handwritten note:  $2(2x-13)$*

**HABITS OF MIND**

**Make Sense and Persevere** Explain why it does not make sense to add the denominators when adding rational numbers. Use numerical fractions to support your thinking. **MP.1**

**EXAMPLE 2** **Try It!** Identify the Least Common Multiple of Polynomials

2. Find the LCM for each set of expressions. **LCM**

*Factor*

a.  $x^3 + 3x^2 + 9x + 27, x^2 - 4x - 21$

$x^2$	$x^3$	$3x^2$
9	$9x$	27

*Handwritten notes:  $(x-7)(x+3)$ ,  $(x^2+9)(x+3)$*

**LCM**  
 $(x+3)(x^2+9)(x-7)$

b.  $10x^2 - 10y^2, 15x^2 - 30xy + 15y^2, x^2 + 3xy + 2y^2$

*Handwritten notes:  $10(x^2-y^2) = 10(x-y)(x+y)$ ,  $15(x^2-2xy+y^2) = 15(x-y)(x-y)$ ,  $(x+2y)(x+y)$*

**LCM**  
 $2 \cdot 3 \cdot 5 (x-y)(x+y)(x-y)(x+2y)$

**EXAMPLE 3** **Try It!** Add Rational Expressions With Unlike Denominators

3. Find the sum.

a.  $\frac{x+6}{(x-2)(x+2)} + \frac{2}{(x^2-4)(x-5x+6)}$

*Handwritten notes: LCD  $(x-2)(x+2)(x-3)$*

$$\rightarrow \frac{(x+6) \cdot (x-3)}{(x-2)(x+2)(x-3)} + \frac{2 \cdot (x+2)}{(x-3)(x-2)(x+2)}$$

$$\rightarrow \frac{x^2+3x-18+2x+4}{(x-2)(x+2)(x-3)}$$

$$\rightarrow \frac{(x+7)(x-2)}{(x-2)(x+2)(x-3)} \rightarrow \frac{(x+7)}{(x+2)(x-3)}$$

b.  $\frac{2x}{(3x+4)} + \frac{4x^2-11x-12}{(6x^2+5x-4)}$

*Handwritten notes: LCD  $(3x+4)(2x-1)$*

$$\rightarrow \frac{2x \cdot (2x-1)}{(3x+4)(2x-1)} + \frac{4x^2-11x-12}{(3x+4)(2x-1)}$$

$$\rightarrow \frac{4x^2-2x+4x^2-11x-12}{(3x+4)(2x-1)}$$

$$\rightarrow \frac{8x^2-13x-12}{(3x+4)(2x-1)}$$

*Handwritten note:  $8x^2-13x-12$*

**EXAMPLE 4** **Try It!** Subtract Rational Expressions

$$\frac{\text{LCD}}{3x \cdot 2 \cdot x} \\ 6x^2$$

4. Simplify.

a.  $\frac{1}{3x} + \frac{1}{6x} - \frac{1}{x^2}$

$\frac{1}{3x} \cdot \frac{2x}{2x} + \frac{1}{6x} \cdot \frac{2x}{2x} - \frac{1}{x^2} \cdot \frac{6}{6} = \frac{2x+2x-6}{6x^2} = \frac{4x-6}{6x^2} = \frac{2(2x-3)}{2 \cdot 3x^2} = \frac{2x-3}{3x^2}$

b.  $\frac{3x-5}{x^2-25} - \frac{2x+10}{x-5}$

$\frac{(3x-5)}{(x-5)(x+5)} - \frac{(2x+10)}{(x-5)} \cdot \frac{(x+5)}{(x+5)} = \frac{3x-5-2x+10}{(x-5)(x+5)} = \frac{x+5}{(x-5)(x+5)} = \frac{1}{x-5}$

**HABITS OF MIND**

**Communicate Precisely** How does finding the LCM of two or more polynomials help you to add and subtract rational expressions? © MP.6

Humph....

**EXAMPLE 5** **Try It!** Find a Rate

5. On the way to work Juan carpools with a fellow co-worker, and then takes the city bus back home in the evening. The average speed of the 20-mi trip is 5 mph faster in the carpool. Write an expression that represents Juan's total travel time.

**HABITS OF MIND**

**Construct Arguments** Does Juan spend more time in the carpool or riding the bus? How do you know? © MP.3

**EXAMPLE 6** **Try It!** Simplify a Compound Fraction

$\frac{\frac{x+1}{3} + \frac{4}{x-1}}{\frac{3}{x^2-1} + \frac{4}{x-1}}$

$\frac{\frac{(x+1)(x-1)}{3(x-1)} + \frac{4 \cdot 3}{(x-1) \cdot 3}}{\frac{x^2-1+12}{3(x-1)} + \frac{x^2+11}{3(x-1)}}$

$\frac{\frac{1}{x-1} \cdot \frac{x^2+11}{3(x-1)}}{\frac{x^2+11}{3(x-1)}}$

6. Simplify.

a.  $\frac{\frac{1}{x+1} + \frac{4}{x-1}}{\frac{3}{x^2+11}}$

$\frac{1 + \frac{4(x+1)}{x-1}}{\frac{3}{x^2+11}}$

b.  $\frac{\frac{x-1}{x} \cdot \frac{x}{x+2}}{\frac{x-1}{x} \cdot \frac{x}{x+2}}$

$\frac{x-1}{x+2}$

**HABITS OF MIND**

**Reason** Edwin multiplied the top and bottom of the fraction in problem 6 part (a) by  $\frac{3}{x^2+11} + \frac{x}{x-1}$ . Will this technique work to simplify the compound fraction? Explain. © MP.2

**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How do you rewrite rational expressions to find sums and differences?

2. **Vocabulary** In your own words, define compound fraction and provide an example of one.

3. **Error Analysis** A student added the rational expressions as follows:  
 $\frac{5x}{x+7} + \frac{7}{x} = \frac{5x}{x+7} + \frac{7(7)}{x+7} = \frac{5x+49}{x+7}$   
 Describe and correct the error the student made. **MP.3**

4. **Construct Arguments** Explain why, when stating the domain of a sum or difference of rational expressions, not only should the simplified sum or difference be considered but the original expression should also be considered. **MP.3**

5. **Make Sense and Persevere** In adding or subtracting rational expressions, why is the L in LCD significant? **MP.1**

**Do You KNOW HOW?**

6. Find the sum of  $\frac{3}{x+1} + \frac{11}{x+1}$

$$\frac{3+11}{LCD} = \frac{14}{x+1}$$

Find the LCM of the polynomials.

7.  $x^2 - y^2$  and  $x^2 - 2xy + y^2$   
 diff of squares      PSF  
 $(x-y)(x+y)$        $(x-y)(x-y)$        $\frac{LCD?}{(x-y)(x+y)(x-y)}$

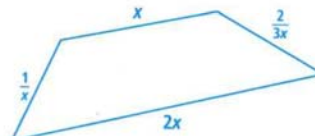
8.  $5x^3y$  and  $15x^2y^2$   
 $5 \times x \times x \times y$        $3 \times 5 \times x \times x \times y \times y$        $3 \times 5 \times x \times x \times y \times y$   
 $\frac{LCD?}{15x^3y^2}$

Find the sum or difference.

9.  $\frac{3x}{4y^2} - \frac{y}{10x}$   
 $\frac{3x}{4y^2} \cdot \frac{5x}{5x} - \frac{y}{10x} \cdot \frac{2y^2}{2y^2}$   
 $\frac{15x^2 - 2y^3}{20xy^2}$

10.  $\frac{(3y+2)}{(3y^2-2y-8)} + \frac{7}{(3y^2+y-4)}$   
 $\frac{(3y+4)(y-2)}{(3y+4)(y-2)} + \frac{7}{(3y+4)(y-1)}$   
 $\frac{9y^2-9y+2y-2+7y+7}{(3y+4)(y-2)(y-1)}$   
 $\frac{9y^2-16}{(3y+4)(y-2)(y-1)}$

11. Find the perimeter of the quadrilateral in simplest form. distance around



$$\frac{1}{x} + x + \frac{2}{3x} + \frac{2x}{3x}$$

$$\frac{3+3x^2+2+6x^2}{3x} \rightarrow \frac{9x^2+5}{3x}$$