

From 4.4: Rational Expressions → Find the LCD

CRITIQUE & EXPLAIN

Nicky and Tavon used different methods to solve the equation $\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$

Nicky	Tavon
$\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$	LCD $\frac{1}{2}x + \frac{2}{5} = \frac{9}{10}$
$\frac{1}{2}x = \frac{9}{10} - \frac{2}{5}$	$10(\frac{1}{2}x + \frac{2}{5} = \frac{9}{10})$
$\frac{1}{2}x = \frac{5}{10}$	$5x + 4 = 9$
$x = 1$	$5x = 5$
The solution is 1.	$x = 1$ The solution is 1.

manipulate fractions

multiply by LCD

4-5
Solving Rational Equations

→ Eliminate the Denominator
• Multiply by the LCD
→ Numerators

A. Explain the different strategies that Nicky and Tavon used and the advantages or disadvantages of each.

See above... Tavon's: no fractions...

B. Did Nicky use a correct method to solve the equation? Did Tavon?

• Both correct!

C. Use Structure Why might Tavon have chosen to multiply both sides of the equation by 10? Could he have used another number? Explain. MP.7

Yes... any common denominator...

HABITS OF MIND

Reason If Tavon had multiplied both sides of the equation by 100, would his answer have been 10 times as much? Explain. MP.2

No... The answer will reduce to 10...

Proportion *means cross multiply*
 $\frac{a}{b} = \frac{c}{d}$ $\leftrightarrow ad = bc$
extremes

EXAMPLE 1 Try It! Solve a Rational Equation

1. What is the solution to each equation?

a. $\frac{2}{x+5} = \frac{4}{1}$

$$2(1) = (x+5)4$$

$$2 = 4x + 20$$

$$\begin{array}{r} -20 \\ -18 = 4x \end{array}$$

$$\frac{-18}{4} = \frac{4x}{4}$$

$$\frac{-9}{2} = x$$

b. $\frac{1}{x-7} = 2$

$$1(1) = (x-7)2$$

$$1 = 2x - 14$$

$$\begin{array}{r} +14 \\ 15 = 2x \end{array}$$

$$\frac{15}{2} = \frac{2x}{2}$$

$$\frac{15}{2} = x$$

HABITS OF MIND

Critique Reasoning For part (b), Kaitlyn wrote $1 = 2x - 7$, then $8 = 2x$, and $4 = x$. Is she correct? Explain. **MP3**

EXAMPLE 2 Try It! Solve a Work-Rate Problem \rightarrow Individually & together

2. It takes 12 hours to fill a pool with two pipes, where the water in one pipe flows three times as fast as the other pipe. How long will it take the slower pipe to fill the pool by itself?

Let $x =$
1st pipe
Let $y =$
2nd pipe

$$\frac{1}{x} + \frac{3}{x} = \frac{1}{12}$$

time *3 times* *time*



$$\frac{4}{x} = \frac{1}{12}$$

$$4(12) = x$$

$$48 = x$$

hours.

Pipes $\frac{48}{3} = 16$

HABITS OF MIND

Reason In a Work-Rate problem, explain why you can't average the individual rates to determine how long it will take to complete a job together. **MP2**

* Mult by LCD

• numerator only

EXAMPLE 3 Try It! Identify an Extraneous Solution

3. What is the solution to the equation $\frac{1}{x+2} + \frac{1}{x-2} = \frac{4}{(x+2)(x-2)}$?

→ makes the value of denominator 0 ...

$$\frac{1}{(x+2)(x-2)} \cdot (x+2)(x-2) + \frac{1}{(x-2)} \cdot (x+2)(x-2) = \frac{4}{(x+2)(x-2)} \cdot (x+2)(x-2)$$

$$\frac{1}{x-2} + \frac{x+2}{x-2} = 4$$

$$\frac{1 + x + 2}{x-2} = 4$$

$$\frac{x+3}{x-2} = 4$$

$$x+3 = 4(x-2)$$

$$x+3 = 4x-8$$

$$-3x = -11$$

$$x = \frac{11}{3}$$

no solution

EXAMPLE 4 Try It! Solve Problems With Extraneous Solutions

4. What are the solutions to the following equations?

a. $x + \frac{6}{x-3} = \frac{2x}{x-3}$ $x=3$

mult LCD $(x-3)$

$$x(x-3) + 6 = 2x$$

$$x^2 - 3x + 6 = 2x$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

extraneous solution $x=3$ $x=2$



b. $\frac{x^2}{x+5} = \frac{25}{x+5}$ $x=5$

$$x^2 = 25$$

$$x^2 - 25 = 0$$

$$(x-5)(x+5) = 0$$

extraneous solution $x=5$ $x=-5$

HABITS OF MIND

Communicate Precisely What is an extraneous solution? © MP.6

→ values that make denominators zero...

EXAMPLE 5 Try It! Solve a Rate Problem

5. Three people are planting tomatoes in a community garden. Marta takes 50 minutes to plant the garden alone. Benito takes x minutes and Tyler takes $x + 15$ minutes. If three people take 20 minutes to finish the garden, how long would it have taken Tyler alone?



HABITS OF MIND

Make Sense and Persevere What does the fraction $\frac{1}{50}$ mean with regards to Marta? © MP.1



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve rational equations and identify extraneous solutions?

2. **Vocabulary** Write your own example of a rational equation that, when solved, has at least one extraneous solution.

3. **Error Analysis** A student solved the rational equation as follows:

$$\frac{1}{2x} - \frac{2}{5x} = \frac{1}{10x} - 3; x = 0$$

Describe and correct the error the student made. © MP.3

4. **Construct Arguments** Yuki says, "You can check the solution(s) of rational equations in any of the steps of the solution process." Explain why her reasoning is incorrect. © MP.3

Do You KNOW HOW?

Solve. *Proportion?*

$$5. \frac{4}{x+6} = \frac{2}{7}$$

$$4(7) = 2(x+6)$$

$$4 = 2x+12$$

$$-8 = 2x$$

$$\boxed{-4 = x}$$

$$6. \frac{x^2}{x+3} = \frac{9}{x-3}$$

$$x^2 - 9 = 0$$

$$(x-3)(x+3) = 0$$

$$\boxed{x=3} \quad \cancel{x=-3}$$

extraneous

7. Organizing given information into a table can be helpful when solving rate problems. Use this table to solve the following problem.

	Distance	Rate	Time
Upstream			
Downstream			

The speed of a stream is 4 km/h. A boat can travel 6 km upstream in the same time it takes to travel 12 km downstream. Find the speed of the boat in still water.

