Activity

5-1 nth Roots, Radicals, and Rational Exponents

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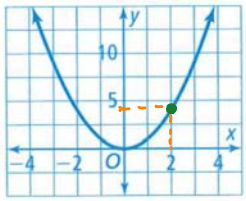
EXPLORE & REASON

The graph shows $y = x^2$. parabola



A. Find *all* possible values of x or y so that the point is on the graph.

(a) (2, x 4) (b) (3, 9) (c) (−3, 9) (d) (5, 25)

(e) (2, 4) (f) (4i, −16) (g) (√7, 7) (h) (√5, 5)

? Imaginary • no real soln.

B. **Communicate Precisely** Write a precise set of instructions that show how to find an approximate value of $\sqrt{13}$ using the graph. MP.6

3^2 9 4^2 16 (3

y → between 3 & 4 → 3.5ish 3.6ish

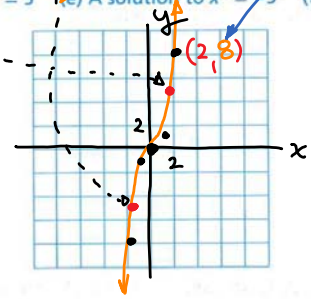
C. Draw a graph of $y = x^3$. Use the graph to approximate each value.

(a) $\sqrt[3]{5}$ y=5 → x≈1.71ish

(b) $\sqrt[3]{-5}$ $\sqrt[3]{y}=x$ → x≈−1.71ish y=−5

(c) $\sqrt[3]{8}$ y=8 → x=2

(d) A solution to $x^3 = 5$ (e) A solution to $x^3 = -5$ (f) A solution to $x^3 = 8$



Review

Rational Exponents (aka Fraction Powers)

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ (m ← power, n ← roots)

fraction power form | radical form

ex) $5^{\frac{2}{3}} = \sqrt[3]{5^2}$ or $\left(\sqrt[3]{5}\right)^2$

index (root) $\sqrt[3]{141}$ radicand, radical

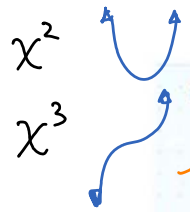
HABITS OF MIND

Look for Relationships How is $\sqrt[6]{5}$ related to $\sqrt[3]{5}$? MP.7

$5^{\frac{1}{6}}$ $\sqrt[6]{5}$ vs. $\sqrt[3]{5}$

$\sqrt[2]{\sqrt[3]{5}}$ → $\left(5^{\frac{1}{3}}\right)^{\frac{1}{2}}$

radical; cube root; square root



Notes

real roots
• x-ints/solutions/zeros

Assess

EXAMPLE 1 **Try It! Find All Real *n*th Roots**

1. Find the specified roots of each number.

a. real fourth roots of 81

$x^4 = 81$

$x^4 - 81 = 0$ diff of squares

$(x^2-9)(x^2+9) = 0$

$(x-3)(x+3)(x^2+9) = 0$

$x=3$ | $x=-3$ | 2 imag → $\pm 3i$

b. real cube roots of 64

$x^3 = 64$

$x^3 - 64 = 0$

$(x-4)(x^2+4x+16) = 0$

$x=4$ | 2 imag because $b^2-4ac \to 4^2-4(1)(16) \to 16-64 \to -48$

ex) What are the real cube roots of 125.

$x^3 = 125$

GC: $x = \sqrt[3]{125}$, $x=5$

diff of cubes $x^3 - 125 = 0$

$a=x$ $b=5$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$(x-5)(x^2+5x+25)=0$

$x-5=0$ | $x^2+5x+25=0$ → x: 2 imag roots → discriminant: -75

b^2-4ac

$5^2-4(1)(25)$

$x=5$

EXAMPLE 2 **Try It! Understand Rational Exponents** aka fraction powers

2. Explain what each fractional exponent means, then evaluate.

a. $25^{\frac{1}{2}}$ (power, root; rat'l exp form)

$25^{\frac{1}{2}}$

$\sqrt[2]{25^1} \to 5$ (radical form)

b. $32^{\frac{2}{5}}$

$32^{\frac{2}{5}}$ → radical form

$\sqrt[5]{32^2} \to \sqrt[5]{2\cdot2\cdot2\cdot2\cdot2 \cdot 2\cdot2\cdot2\cdot2\cdot2}$ (power, index)

$2 \cdot 2 \to 4$

HABITS OF MIND

Generalize What is true about the denominators of fractional exponents in which absolute value must be considered? MP.8

$100^{\frac{1}{3}}$ (odd) vs. $75^{\frac{1}{2}}$ (even : roots)

even $\sqrt{\text{neg \#}}$

EXAMPLE 3 **Try It! Evaluate Expressions With Rational Exponents**

3. What is the value of each expression? Round to the nearest hundredth if necessary.

a. $-(16^{\frac{3}{4}})$ (radical)

$-(16^{\frac{3}{4}}) \to -(2^4)^{\frac{3}{4}}$ OR $\sqrt[4]{16\cdot16\cdot16}$

$\to -(2)^3 \to -8$

$-2\cdot2\cdot2 \to -8$

b. $\sqrt[5]{3.5^4}$

GC: $3.54 \boxed{\wedge} (4 \div 3)$

or $5 \boxed{\sqrt[x]{}} 3.5 \boxed{\wedge} 4$

2.72

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Notes

Assess

EXAMPLE 4 Try It! Simplify *n*th Roots

4. Simplify each expression.

a. $\sqrt[3]{-8a^3b^9}$

b. $\sqrt[4]{256x^{12}y^{24}}$

odd root — triples — odd power

even root : ... what if variables are negative ?

$(-8a^3b^9)^{\frac{1}{3}}$

$(-8)^{\frac{1}{3}}(a^3)^{\frac{1}{3}}(b^9)^{\frac{1}{3}}$

$\sqrt[3]{-8}\ a^1b^3$

$-2 \cdot -2 \cdot -2\ a\,a\,a\,b\,b\,b\,b\,b\,b\,b\,b\,b$

$-2ab^3 \rightarrow -2ab^3$

$4 \cdot 4 \cdot 4 \cdot 4$

$x^4 x^4 x^4$

$y^4 y^4 y^4 y^4 y^4 y^4$

$4|x^3|y^6$ or $\pm 4x^3y^6$ — neg ?

HABITS OF MIND

Make Sense and Persevere What is an example of a variable expression that has both a cube root and a fourth root which can be simplified to an expression without a radical? MP.1

EXAMPLE 5 Try It! Use *n*th Roots to Solve Equations

5. a. Solve the equation $5x^3 = 320$.

b. Solve the equation $2p^4 = 162$.

odd — even ±

PE MD AS — Simplify — SOLVE

$\frac{5x^3}{5} = \frac{320}{5}$

$x^3 = 64$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

$\frac{2p^4}{2} = \frac{162}{2}$

$p^4 = 81$

fraction powers

$p^{\frac{4}{1}} = 81$

inverse of $\frac{4}{1}$

$(p^{\frac{4}{1}})^{\frac{1}{4}} = (81)^{\frac{1}{4}}$

$p = \pm$

$\sqrt[4]{p^4} = \pm\sqrt[4]{81}$

$p = \pm\sqrt[4]{81}$

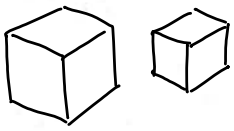
$p = \pm 3$

81

$\pm(3^4)^{\frac{1}{4}} = \pm 3$

EXAMPLE 6 Try It! Use *n*th Roots to Solve Problems

6. One cube has an edge length 3 cm shorter than the edge length of a second cube. The volume of the smaller cube is 200 cm^3. What is the volume of the larger cube?



$V = s^3$ Let x = side of larger cube..

$200 = (x-3)^3$

$\sqrt[3]{200} \quad \sqrt[3]{(x-3)^3}$

$\sqrt[3]{200} = x - 3$

$\sqrt[3]{200} + 3 = x$

$= x \approx 8.84$ cm

$V = s^3$

$V = (8.84)^3$

$V = 692.7$ cm^3

HABITS OF MIND

Communicate Precisely What are the steps necessary to solve the equation $ax^n = b$? MP.6

Assess

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How are exponents and radicals used to represent roots of real numbers?

2. **Error Analysis** Kaitlyn said $\sqrt[3]{10} = 10^3$. Explain Kaitlyn's error. MP.3

3. **Vocabulary** In the radical expression $\sqrt[5]{125}$, what is the index? What is the radicand?

4. **Reason** Why is $75^{\frac{3}{5}}$ equal to $(75^{\frac{1}{5}})^3$? MP.2

5. **Critique Arguments** Anastasia said that $(x^8)^{\frac{1}{4}} = \frac{x^8}{x^4} = x^4$. Is Anastasia correct? Explain. MP.3

6. **Make Sense and Persevere** Is it possible for a rational exponent be an improper fraction? Explain how $27^{\frac{4}{3}}$ is evaluated or why it cannot be evaluated. MP.1

Do You KNOW HOW?

Write each expression in radical form.

7. $a^{\frac{1}{5}}$ $\sqrt[5]{a}$

8. $7^{\frac{2}{3}}$ $\sqrt[3]{7^2}$

Write each expression in exponential form.

9. $\sqrt[3]{b}$ $b^{\frac{1}{3}}$

10. $\sqrt[4]{p^7}$ $p^{\frac{7}{4}}$

11. How many real third roots does 1,728 have?
$x^3 = 1728$, $x^3 - 1728 = 0$ → $(x-12)(x^2+12x+144)$, $x = 12$; 1 real, 2 imag; one real root

12. How many real sixth roots does 15,625 have?
$x^6 = 15625$, $x^6 - 15625 = 0$ → $(x^2-25)(x^4+25x^2+625)$, $(x-5)(x+5)(x^4+25x^2+625)$; $x = 5, -5$ → two real roots

13. Solve the equation $4x^3 = 324$
$\frac{4x^3}{4} = \frac{324}{4}$, $x^3 = 81$, $x = \sqrt[3]{81}$, $x = 3\sqrt[3]{3}$

14. Solve the equation $2x^4 = 2{,}500$
$2x^4 = 2500$, $x^4 = 1250$, $x = \pm\sqrt[4]{1250}$; $125 = 5 \cdot 25$, $25 = 5 \cdot 5$, $10 = 5 \cdot 2$ → $x = \pm 5\sqrt[4]{2}$

Simplify each expression.

15. $\sqrt[3]{27x^{12}y^6}$ $3 = 3 \cdot 3 \cdot 3$; $3x^{\frac{12}{3}}y^{\frac{6}{3}} \rightarrow 3x^4y^2$

16. $\sqrt[5]{-32x^5y^{30}}$ $-2 = -2 \cdot -2 \cdot -2 \cdot -2 \cdot -2$; $-2x^{\frac{5}{5}}y^{\frac{30}{5}} \rightarrow -2xy^6$

17. A snow globe is packaged in a cubic container that has volume 64 in.^{3} A large shipping container is also a cube, and its edge length is 8 inches longer than the edge length of the snow globe container. How many snow globes can fit into the larger shipping container?