Activity

# 5-2 Properties of Exponents and Radicals

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## CRITIQUE & EXPLAIN

Olivia was practicing evaluating and simplifying expressions. Her work for three expressions is shown.

1. $24^2 = 400 + 16 = \cancel{4}16$ 576
2. $3^6 = 9(\cancel{2}7) = 270 - 27 = 243$ 729
3. $\sqrt{625} = \sqrt{400} + \sqrt{225} = 20 + 15 = 35$

A. Is Olivia's work in the first example correct? Explain your thinking.

$(20+4)^2 \neq (20^2+4^2)$

illegal

B. Is Olivia's work in the second example correct? Explain your thinking.

$3^6 = 9(27) = 270-27 \rightarrow 27(\underbrace{10-1}_{9})$

$3^2 \cdot 3^3$ illegal

$27 \cdot 27$

C. Is Olivia's work in the third example correct? Explain your thinking.

$\sqrt{625} = \sqrt{400} + \sqrt{225}$

illegal

$\sqrt{400+225}$

D. Make Sense and Persevere What advice would you give Olivia on simplifying expressions? MP.1

Product of Powers $a^m \cdot a^n = a^{m+n}$

Power of Powers $(a^m)^n = a^{m \cdot n}$

quotient of powers $\frac{a^m}{a^n} = a^{m-n}$

negative power $a^{-m} = \frac{1}{a^m}$

zero power $a^0 = 1$

**HABITS OF MIND**

**Construct Arguments** You know that $3^2 + 4^2 = 5^2$. Does $\sqrt{3^2} + \sqrt{4^2} = \sqrt{5^2}$? If not, how could you rewrite the equation using radicals so that it is true? MP.3

$a^2+b^2=c^2$

illegal

$\sqrt{3^2+4^2} = \sqrt{5^2}$

• Reduced Radical Form

**EXAMPLE 1** **Try It!** Use Properties of Exponents

1. How can you rewrite each expression using the properties of exponents?

a. $\left(\frac{3}{32^{\frac{2}{5}}}\right)^{\frac{1}{2}}$ apply power

$\frac{3^{\frac{1}{2}}}{\left(32^{\frac{2}{5}}\right)^{\frac{1}{2}}} \to \frac{3^{\frac{1}{2}}}{32^{\frac{1}{5}}} \to \frac{3^{\frac{1}{2}}}{2} \to \boxed{\frac{\sqrt{3}}{2}}$

power of a power

square root

$\sqrt[5]{22222}$

b. $2a^{\frac{1}{3}}(ab^{\frac{1}{2}})^{\frac{2}{3}}$ apply power

$2a^{\frac{1}{3}} \cdot a^{\frac{2}{3}}(b^{\frac{1}{2}})^{\frac{2}{3}}$

$\to 2a^{\frac{1}{3}}a^{\frac{2}{3}}b^{\frac{1}{3}}$

$\to 2a^{\frac{1}{3}+\frac{2}{3}}b^{\frac{1}{3}}$

$\to 2a^1b^{\frac{1}{3}} \to \boxed{2a\sqrt[3]{b}}$ cube root

**EXAMPLE 2** **Try It!** Use Properties of Exponents to Rewrite Radicals

2. How can you rewrite each expression?

a. $\sqrt[4]{81a^8b^5}$

$3\,3\,3\,3$   $a^2a^2a^2a^2$   bbbbb

leftover

$\boxed{3a^2b\sqrt[4]{b}}$

b. $\sqrt[3]{\frac{x^4y^2}{125x}}$

$5\,5\,5$

xxx

$\frac{\sqrt[3]{x^3y^2}}{5}$ leftover

$\to \boxed{\frac{x}{5}\sqrt[3]{y^2}}$

or $\frac{x\sqrt[3]{y^2}}{5}$

**HABITS OF MIND**

Make Sense and Persevere What do you have to check to be sure that an expression is in simplest radical form? MP.1

**EXAMPLE 3** **Try It!** Rewrite the Product or Quotient of a Radical

blah...

3. What is the reduced radical form of each expression?

root

radicals   denominator

monomials

a. $\sqrt[5]{\frac{7}{16x^3}}$

$\sqrt[5]{\frac{7}{2222xxx}} \cdot \sqrt[5]{\frac{2 \cdot x \cdot x}{2 \cdot x \cdot x}}$

$\to \sqrt[5]{\frac{7 \cdot 2xx}{22222\,xxxxx}}$

$\to \frac{\sqrt[5]{14x^2}}{2x}$

b. $\sqrt[4]{27x^2} \cdot \sqrt{3x}$

$(27x^2)^{\frac{1}{4}}(3x)^{\frac{1}{2}}$ → rationalize the denominator...

$\to (27x^2)^{\frac{1}{4}}(3x)^{\frac{2}{4}}$ undoing power of a power

$\to (27x^2)^{\frac{1}{4}}((3x)^2)^{\frac{1}{4}}$ undoing applying power

$\to (27x^2(3x)^2)^{\frac{1}{4}}$

$\to (27x^2 9x^2)^{\frac{1}{4}} \to (243x^4)^{\frac{1}{4}} \to 243^{\frac{1}{4}}x^{\frac{4}{4}}$

$\rightarrow (27x^2\, 9x^2)^{\frac{1}{4}} \rightarrow (243x^4)^{\frac{1}{4}} \rightarrow 243^{\frac{1}{4}} x^{\frac{4}{4}}$

$\rightarrow x\sqrt[4]{243} \rightarrow x\sqrt[4]{3\,3\,\cancel{3\cdot 3}\,3} \rightarrow \boxed{3x\cdot\sqrt[4]{3}}$

Notes

EXAMPLE 4

## Try It! Add and Subtract Radical Expressions

same

4. How can you rewrite each expression in a simpler form?

a. $\sqrt[3]{2{,}000} + \sqrt{2} - \sqrt[3]{128}$

2 1000

10 10 10

2 2 2 2 2 2 2

$\rightarrow 10\sqrt[3]{2} + \sqrt{2} - 2\cdot2\sqrt[3]{2}$

$\rightarrow 10\sqrt[3]{2} + \sqrt{2} - 4\sqrt[3]{2}$

$\rightarrow \boxed{6\sqrt[3]{2} + \sqrt{2}}$

b. $\sqrt{20} - \sqrt{600} - \sqrt{125}$

2 2 5, 2 3 10 10, 5 5 5

$\rightarrow 2\sqrt{5} - 10\sqrt{6} - 5\sqrt{5}$

$\rightarrow \boxed{-3\sqrt{5} - 10\sqrt{6}}$

HABITS OF MIND

Critique Reasoning Divit says that you can simplify the product of any two radical expressions, but not necessarily the sum. Is he correct? Give an example. MP.3

EXAMPLE 5

## Try It! Multiply Binomial Radical Expressions

$1\sqrt{6} \cdot 5\sqrt{1}$

5. Multiply.

a. $(x - \sqrt{10})(x + \sqrt{10})$

Conjugates

FOIL

|  | $x$ | $+\sqrt{10}$ |
|---|---|---|
| $x$ | $x^2$ | $\cancel{+x\sqrt{10}}$ |
| $-\sqrt{10}$ | $\cancel{-x\sqrt{10}}$ | $-10$ |

$\boxed{x^2 - 10}$

b. $\sqrt{6}(5 + \sqrt{3})$

$5\sqrt{6} + \sqrt{18} \rightarrow \boxed{5\sqrt{6} + 3\sqrt{2}}$

6 3, 3 2

EXAMPLE 6

## Try It! * Rationalize a Binomial Denominator

- Multiply by the Conjugate

$a+\sqrt{b}$ & $a-\sqrt{b}$

6. What is the reduced radical form of each expression?

a. $\dfrac{5 - \sqrt{2}}{2 - \sqrt{3}} \cdot \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})}$ FOIL

$\rightarrow \dfrac{10 + 5\sqrt{3} - 2\sqrt{2} - \sqrt{6}}{4 + \cancel{2\sqrt{3}} - \cancel{2\sqrt{3}} - \sqrt{9}}$

3

$\boxed{\dfrac{10 + 5\sqrt{3} - 2\sqrt{2} - \sqrt{6}}{1}}$

b. $\dfrac{-4x}{1 - \sqrt{x}} \cdot \dfrac{(1+\sqrt{x})}{(1+\sqrt{x})}$

$\rightarrow \dfrac{-4x - 4x\sqrt{x}}{1 + \cancel{\sqrt{x}} - \cancel{\sqrt{x}} - x}$

$\rightarrow \boxed{\dfrac{-4x - 4x\sqrt{x}}{1 - x}}$

HABITS OF MIND

Reason Is the product of two irrational binomials always irrational? Explain. MP.2

No... product of conjugates $\rightarrow$ rational

Assess

## Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can properties of exponents and radicals be used to rewrite radical expressions?

2. **Vocabulary** How can you determine if a radical expression is in reduced form?

3. **Use Structure** Explain why $(-64)^{\frac{1}{3}}$ equals $-64^{\frac{1}{3}}$ but $(-64)^{\frac{1}{2}}$ does not equal $-64^{\frac{1}{2}}$. MP.7

4. **Error Analysis** Explain the error in Julie's work in rewriting the radical expression. MP.3

$\sqrt{-3} \cdot \sqrt{-12} = \sqrt{-3(-12)} = \sqrt{36} = 6$

## Do You KNOW HOW?

What is the reduced radical form of each expression?

5. $49^{\frac{3}{4}} \cdot 49^{\frac{-1}{4}}$

$49^{\frac{3}{4}+\frac{-1}{4}} \rightarrow 49^{\frac{2}{4}} \rightarrow 49^{\frac{1}{2}} \rightarrow 7$

square root

6. $\left(\frac{a^2b^8}{a^{\frac{1}{3}}}\right)^{\frac{3}{4}}$

$\frac{(a^2)^{\frac{3}{4}}(b^8)^{\frac{3}{4}}}{(a^{\frac{1}{3}})^{\frac{3}{4}}} \rightarrow \frac{a^{\frac{6}{4}}b^6}{a^{\frac{1}{4}}} \rightarrow a^{\frac{6}{4}-\frac{1}{4}}b^6 \rightarrow a^{\frac{5}{4}}b^6$

$a^{\frac{4}{4}+\frac{1}{4}}$

$\sqrt[4]{aaaaa}$

$\boxed{ab^6\sqrt[4]{a}}$

7. $\sqrt[4]{1{,}024x^9y^{12}}$

32 32

xxxx xxxx x

yyyy yyyy yyyy

$\boxed{4x^2|y^3|\sqrt[4]{4x}}$

even root

abs

8. $\sqrt[3]{\frac{4}{9m^2}}$

need triples

rationalize denom

$\sqrt[3]{\frac{22 \cdot 3m}{33mm \cdot 3m}} \rightarrow \boxed{\frac{\sqrt[3]{12m}}{3m}}$

9. $\sqrt{63} - \sqrt{700} - \sqrt{112}$

$\rightarrow 3\sqrt{7} - 10\sqrt{7} - 4\sqrt{7} \rightarrow \boxed{-11\sqrt{7}}$

10. $\sqrt{5}(6 + \sqrt{2})$

$\boxed{6\sqrt{5} + \sqrt{10}}$

11. $\frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} \rightarrow \frac{3\sqrt{6}}{6} \rightarrow \boxed{\frac{\sqrt{6}}{2}}$

rationalize

12. $\frac{\sqrt{7}}{(\sqrt{5} + 3)} \cdot \frac{(\sqrt{5}-3)}{(\sqrt{5}-3)} \rightarrow \frac{\sqrt{35} - 3\sqrt{7}}{5 - 9} \rightarrow \boxed{\frac{\sqrt{35}-3\sqrt{7}}{-4}}$

rationalize: mult by conjugate

FOIL