

5-3

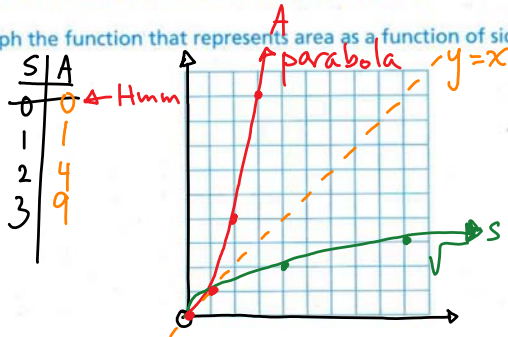
Graphing Radical Functions

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EXPLORE & REASON

Consider the formula for the area of a square: $A = s^2$.

A. Graph the function that represents area as a function of side length.



* $S = \sqrt{A}$
 • swap axes...
 Hmm...

B. On the same set of axes, graph the function that represents side length as a function of area.

$$A = s^2$$

$$\sqrt{A} = \sqrt{s^2}$$

$$\sqrt{A} = s$$

C. Look for Relationships How are the two graphs related? © MP.7

They are inverses of one another...

HABITS OF MIND

Communicate Precisely What is the domain and range of each function? © MP.6

$$A = s^2$$

$$d: \{s | s > 0\}$$

$$r: \{A | A > 0\}$$

$$s = \sqrt{A}$$

$$d: \{A | A > 0\}$$

$$r: \{s | s > 0\}$$

square
 $y = x^2$
 $\sqrt{\quad}$ \downarrow $\pm\sqrt{\quad}$



square root
 $y = \sqrt{x}$



Notes

inverse

EXAMPLE 1

Try It! Graph Square Root and Cube Root Functions

1. Graph the following functions. What is the domain and range of each function? Is the function increasing or decreasing?

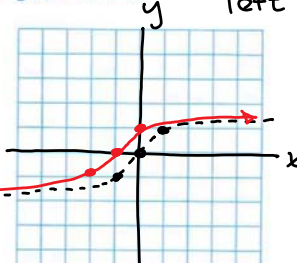
a. $f(x) = \sqrt{x-5}$ $h: 5$ right

b. $g(x) = \sqrt[3]{x+1}$ $h: -1$ left



$d: \{x \mid x \geq 5\}$
 $r: \{y \mid y \geq 0\}$

→ increasing



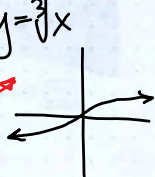
$d: \{x \mid \mathbb{R}\}$
 $r: \{y \mid \mathbb{R}\}$

→ increasing

cube
 $y = x^3$
 $\sqrt[3]{\quad}$ \downarrow $\sqrt[3]{\quad}$



cube root
 $y = \sqrt[3]{x}$



inverse

Radical Function

$f(x) = a \cdot \sqrt[n]{x-h} + k$

vert stretch/shrink/ reflection

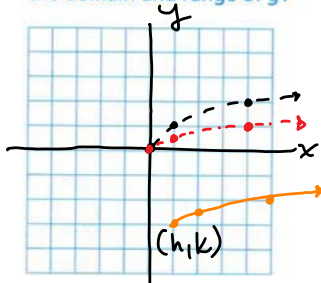
horiz shift

vert shift

EXAMPLE 2 Try It! Graph a Transformation of a Radical Function

2. Graph $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{2}\sqrt{x} - 1 - 3$. What transformations of the graph of f produce the graph of g ? What is the effect of the transformations on the domain and range of g ?

x	y
0	0
1	1
4	2



$a: \frac{1}{2}$ vert shrink
 $h: 1$ shift right
 $k: -3$ shift down

$d: \{x \mid x \geq 1\}$
 $r: \{y \mid y \geq -3\}$

HABITS OF MIND

Use Structure How does the graph of $y = \sqrt{x-a} + b$ compare to the graph of $y = \sqrt{x}$? © MP.7

h k
 → shifting

$$a\sqrt[n]{x-h} + k$$

Notes

Assess

EXAMPLE 3

Try It! Rewrite Radical Functions to Identify Transformations

ex 3) $g(x) = \sqrt{9x} \rightarrow 3\sqrt{x}$

$a: 3$
Vert stretch

3. What transformations of the parent graph of $f(x) = \sqrt{x}$ produce the following graphs?

a. $m(x) = \sqrt{7x - 3.5} - 10$

$a: \sqrt{7}$ Vert stretch
 $h: \frac{1}{2}$ right
 $k: -10$ down

$$\sqrt{7(x - \frac{1}{2})} - 10$$

$$= \sqrt{7} \sqrt{x - \frac{1}{2}} - 10$$

b. $j(x) = -2\sqrt{12x} + 4$

$a: -4\sqrt{3}$ Vert stretch & reflection
 $k: 4$ up

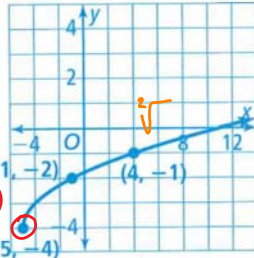
$$-2 \cdot 2\sqrt{3} \cdot \sqrt{x} + 4$$

EXAMPLE 4

Try It! Write an Equation of a Transformation

4. What radical function is represented in each graph below?

a.

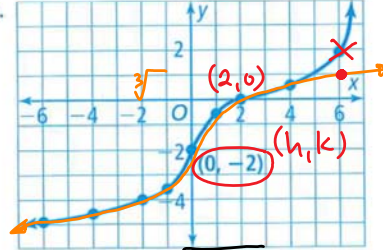


GC or subst a point, $(-1, -2)$
 $-2 = a\sqrt{-1+5} - 4$
 $-2 = a\sqrt{4} - 4$
 $2 = 2a$
 $1 = a$

$$f(x) = a\sqrt{x - h} + k$$

$$f(x) = \sqrt{x+5} - 4$$

b.



$f(x) = a\sqrt[3]{x - h} + k$
Test $(2, 0)$
 $0 = a\sqrt[3]{2 - 0} - 2$
 $0 = a\sqrt[3]{2} - 2$
 $2 = a\sqrt[3]{2}$

$a = \frac{2}{\sqrt[3]{2}}$ Rationalize
 $\frac{2}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{2\sqrt[3]{2}}{2} = \sqrt[3]{4}$

HABITS OF MIND

Model With Mathematics What is an example of a radical function whose domain is $x \geq -3$ and range is $y \geq 2$? © MP.4

→ Square root → $\sqrt[3]{4}\sqrt[3]{x-2}$
or $\sqrt[3]{4x-2}$

$$f(x) = a\sqrt{x-h} + k$$

$a=1$

$$f(x) = \sqrt{x+3} + 2$$

$$f(x) = \sqrt{x+3} + 2$$

EXAMPLE 5

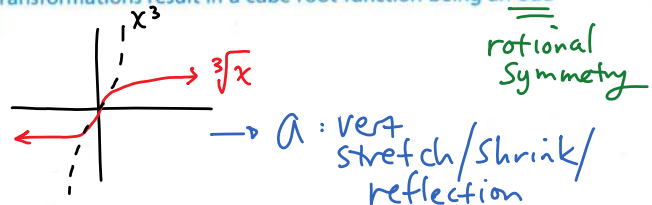
Try It! Interpret a Radical Function Model

5. Use the same function as in Example 3. Suppose Sasha's brother walks through elevations ranging from 5 ft to 48 ft. What are the minimum and maximum distances that he can see?

HABITS OF MIND

Generalize What transformations result in a cube root function being an odd function? © MP.8

$$f(-x) = -f(x)$$



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you use what you know about transformations of functions to graph radical functions?

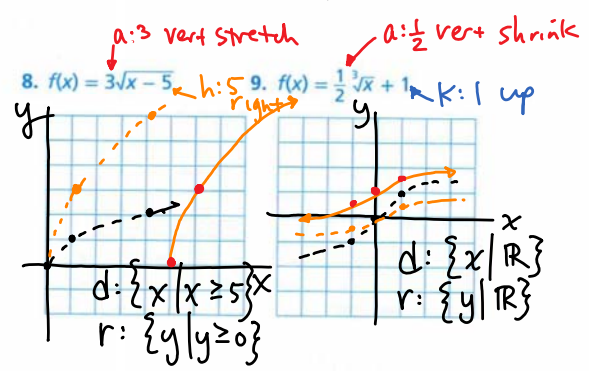
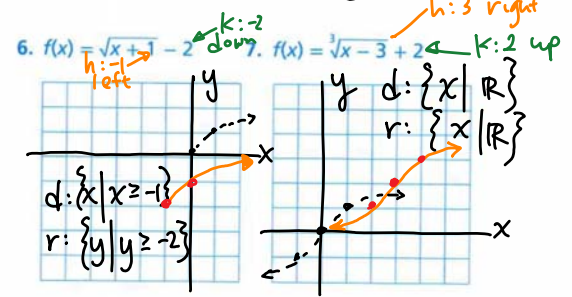
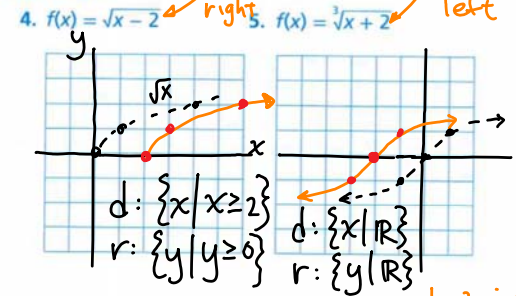


2. **Error Analysis** Parker said the graph of the radical function $g(x) = -\sqrt{x+2} - 1$ is a translation 2 units left and 1 unit down from the parent function $f(x) = \sqrt{x}$. Describe and correct the error. © MP.3

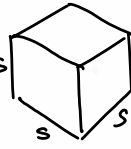
3. **Reason** What effect does a have on the graph of $f(x) = a\sqrt{x}$? © MP.2

Do You KNOW HOW?

Graph each function. Then identify its domain and range.



10. The volume of a cube is a function of the cube's side length. The function can be written as $V(s) = s^3$, where s is the side length and V is the volume.



a. Express a cube's side length as a function of its volume, $s(V)$.

$$V = s^3 \rightarrow \sqrt[3]{V} = \sqrt[3]{s^3}$$

$$\rightarrow \sqrt[3]{V} = s \rightarrow \boxed{s(V) = \sqrt[3]{V}}$$

b. Graph $V(s)$ and $s(V)$. What can you conclude about the graphs? Explain.

They are inverses of each other.

