

SIMPLIFY

PE
MD
AS

SOLVE

OR
Graphing
.732
-2.732

EXPLORE & REASON

A. Solve $3(a+1)^2 + 2 = 11$. Use at least two different methods.

$$\frac{3(a+1)^2}{3} = \frac{9}{3}$$

$$(a+1)^2 = 3$$

$$\sqrt{(a+1)^2} = \pm\sqrt{3}$$

$$a+1 = \pm\sqrt{3}$$

$$a = \frac{-1 \pm \sqrt{3}}{1} \text{ or } \frac{\pm\sqrt{3}-1}{1}$$

$$(a+1)^2 = 3$$

$$(a+1)^2 - 3 = 0$$

$$(a+1-\sqrt{3})(a+1+\sqrt{3}) = 0$$

$$a+1-\sqrt{3}=0 \quad a+1+\sqrt{3}=0$$

$$a = -1+\sqrt{3} \quad a = -1-\sqrt{3}$$

odd root

$$(a+1)(a+1) = 3$$

$$a^2 + 2a + 1 = 3$$

$$a^2 + 2a - 2 = 0$$

Quad formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)}$$

$$\rightarrow \frac{-2 \pm \sqrt{4+8}}{2} \rightarrow \frac{-2 \pm \sqrt{12}}{2}$$

$$\rightarrow \frac{-2 \pm 2\sqrt{3}}{2} \rightarrow -1 \pm \sqrt{3}$$

5-4 Solving Radical Equations

PearsonRealize.com

B. Try each of the methods you used in part (a) to solve $\sqrt[3]{(a+1)} + 2 = 11$.

$$\frac{\sqrt[3]{(a+1)} + 2}{-2} = \frac{11}{-2}$$

OR GC...

$$\sqrt[3]{(a+1)} = 9$$

$$(\sqrt[3]{(a+1)})^3 = (9)^3$$

$$a+1 = 729$$

$$a = 728$$

C. Generalize Which of the methods is better suited for solving an equation with a radical? What problems arise when using the other method? © MP8

... undo operations

HABITS OF MIND

Construct Arguments The squares of two numbers are equal. Does that mean that the two numbers themselves must also be equal? Explain. © MP3

Radical Equations

- Isolate radical(s) & solve

Notes

Assess

EXAMPLE 1 Try It! Solve an Equation With One Radical

1. Solve each radical equation.

a. $\sqrt{x-2} + 3 = 5$

$$\begin{array}{r} \sqrt{x-2} + 3 = 5 \\ \underline{-3 \quad -3} \\ \sqrt{x-2} = 2 \\ (\sqrt{x-2})^2 = (2)^2 \\ x-2 = 4 \\ \underline{+2 \quad +2} \\ x = 6 \end{array}$$

→ Check solutions

b. $\sqrt[3]{x-1} = 2$ *already isolated*

$$\begin{array}{r} \sqrt[3]{x-1} = 2 \\ (\sqrt[3]{x-1})^3 = (2)^3 \\ x-1 = 8 \\ \underline{+1 \quad +1} \\ x = 9 \end{array}$$

EXAMPLE 2 Try It! Rewrite a Formula

2. The speed, v , of a vehicle in relation to its stopping distance, d , is represented by the equation $v = 3.57\sqrt{d}$. What is the equation for the stopping distance in terms of the vehicle's speed?

Isolate d

$$\begin{array}{r} v = 3.57\sqrt{d} \\ \underline{3.57 \quad 3.57} \\ \frac{v}{3.57} = \sqrt{d} \\ \left(\frac{v}{3.57}\right)^2 = (\sqrt{d})^2 \end{array}$$

$$\left(\frac{v}{3.57}\right)^2 = d$$

HABITS OF MIND

Reason Reese solved the equation in 1(a) by first squaring both sides. Is this an appropriate first step? Why or why not? © MP.2

expand to FOIL

$$\begin{array}{r} \sqrt{x-2} + 3 = 5 \\ (\sqrt{x-2} + 3)^2 = (5)^2 \\ (\sqrt{x-2} + 3)(\sqrt{x-2} + 3) = \\ \text{ugh...} \end{array}$$



EXAMPLE 3 Try It! Identify an Extraneous Solution

3. Solve each radical equation. Identify any extraneous solutions.

a. $x = \sqrt{7x+8}$ *already isolated*

$$\begin{array}{r} x = \sqrt{7x+8} \\ (x)^2 = (\sqrt{7x+8})^2 \\ x^2 = 7x+8 \\ \underline{-7x-8 \quad -7x-8} \\ x^2 - 7x - 8 = 0 \\ (x-8)(x+1) = 0 \\ x-8=0 \quad x+1=0 \\ x=8 \quad x=-1 \end{array}$$

extraneous soln..

b. $x+2 = \sqrt{x+2}$

$$\begin{array}{r} x+2 = \sqrt{x+2} \\ (x+2)^2 = (\sqrt{x+2})^2 \\ (x+2)(x+2) = x+2 \\ x^2 + 2x + 2x + 4 = x+2 \\ \underline{-x-2 \quad -x-2} \\ x^2 + 3x + 2 = 0 \\ (x+2)(x+1) = 0 \\ x = -2 \quad x = -1 \end{array}$$

EXAMPLE 4
 $\frac{m}{n} \cdot \frac{n}{m} = 1$
 Inverses
 \rightarrow reciprocal

Try It! Solve Equations With Rational Exponents

4. Solve each equation.

$(x-5)(x+2) = 0$
 $x-5=0 \quad x+2=0$
 $x=5 \quad x=-2$

a. $(x^2 - 3x - 6)^{2/3} - 14 = -6$
 $(x^2 - 3x - 6)^{2/3} = 8$
 $[(x^2 - 3x - 6)^{2/3}]^{3/2} = [8]^{3/2}$
 $x^2 - 3x - 6 = 4$
 $x^2 - 3x - 10 = 0$

b. $(x+8)^{2/3} = (x-2)^3$
 $(x+8)^{2/3} = (x-2)^3$
 $x+8 = (x-2)^3$
 $x+8 = (x-2)^2(x-2)$
 $x+8 = x^2 - 2x - 2x + 4$
 $-x - 18 = x^2 - 4x - 4$
 $0 = x^2 - 5x - 14$
 $0 = (x-7)(x+2)$
 $x=7 \quad x=-2$
 extraneous

EXAMPLE 5

$4x^2 = 16 \cdot 3x$
 $4x^2 = 48x$
 $-48x \quad -48x$
 $4x^2 - 48x = 0$
 mono GCF
 $4x(x-12) = 0$
 $4x=0 \quad x-12=0$
 $x=0 \quad x=12$
 extraneous

Try It! Solve an Equation With Two Radicals

5. Solve each radical equation. Check for extraneous solutions.

a. $\sqrt{x+4} - \sqrt{3x} = -2$
 $\sqrt{x+4} = -2 + \sqrt{3x}$
 $(\sqrt{x+4})^2 = (-2 + \sqrt{3x})^2$
 $x+4 = (-2 + \sqrt{3x})(-2 + \sqrt{3x})$
 $x+4 = 4 - 2\sqrt{3x} - 2\sqrt{3x} + 3x$
 $-3x - 4 = -4\sqrt{3x} - 3x$
 $(-2x)^2 = (-4\sqrt{3x})^2$

b. $\sqrt{15-x} - \sqrt{6x} = -3$
 $\sqrt{15-x} = -3 + \sqrt{6x}$
 $(\sqrt{15-x})^2 = (-3 + \sqrt{6x})^2$
 $15-x = (6x - 3)(6x - 3)$
 $15-x = 6x - 6\sqrt{6x} + 9$
 $-9 - 6x = -6\sqrt{6x} - 9$
 $(-7x+6)^2 = (-6\sqrt{6x})^2$
 $(-7x+6)(-7x+6) = 36 \cdot 6x$
 $49x^2 - 84x + 36 = 216x$
 $-216x$
 $49x^2 - 300x + 36 = 0$
 $(49x-6)(x-6) = 0$
 $x = \frac{6}{49} \quad x = 6$
 extraneous
 or quadratic formula...

HABITS OF MIND

Reason Why are extraneous solutions a possibility for radical equations? © MP.2

EXAMPLE 6

Try It! Solve a Radical Inequality

6. A doctor calculates that a particular dose of medicine is appropriate for an individual whose BSA is less than 1.8. If the mass of the individual is 75 kg, how many centimeters tall can he or she be for the dose to be appropriate?

HABITS OF MIND

Make Sense and Persevere How are the steps for solving a radical inequality different from the steps for solving a radical equation? © MP.1

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How can you solve equations that include radicals or rational exponents?

2. **Construct Arguments** How can you use a graph to show that the solution to $\sqrt[3]{84x+8} = 8$ is 6? © MP3

3. **Vocabulary** Why does solving a radical equation sometimes result in an extraneous solution?

4. **Error Analysis** Neil said that -3 and 6 are the solutions to $\sqrt{3x+18} = x$. What error did Neil make? © MP3

5. **Communicate Precisely** Describe how you would solve the equation $x^{\frac{1}{3}} = n$. How is this solution method to be interpreted if the equation had been written in radical form instead? © MP6

Do You KNOW HOW?

Solve for x.

6. $3\sqrt{x+22} = 21$
 $\sqrt{x+22} = 7$
 $(\quad)^2 (\quad)^2$
 $x+22 = 49$
 $-22 \quad -22$
 $x = 27$

7. $\sqrt[3]{5x} = 25$
 $(\quad)^3 (\quad)^3$
 $5x = 25^3$
 $x = 3125$

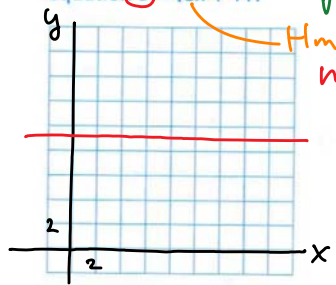
In exercises 8 and 9, find the extraneous solution.

8. $\sqrt{8x+9} = x$
 $(\quad)^2 (\quad)^2$
 $8x+9 = x^2$
 $-8x-9 \quad -8x-9$
 $x^2 - 8x - 9 = 0$
 $(x+9)(x-1) = 0$
 $x = 9 \quad x = 1$ **extraneous**

9. $x = \sqrt{24-2x}$
 $(\quad)^2 (\quad)^2$
 $x^2 = 24-2x$
 $+2x-24 \quad -24+2x$
 $x^2 + 2x - 24 = 0$
 $(x+6)(x-4) = 0$
 $x = -6 \quad x = 4$ **extraneous**

10. Rewrite the equation $y = \sqrt{\frac{x-48}{6}}$ to isolate x.
 $y^2 = \frac{x-48}{6} \rightarrow 6y^2 = x-48 \rightarrow 6y^2 + 48 = x$

11. Use a graph to find the solution to the equation $9 = \sqrt{3x+11}$.
 $9 = \sqrt{3x+11}$
 $81 = 3x+11$
 $-11 \quad -11$
 $70 = 3x$
 $\frac{70}{3} = x$



Solve each equation.

12. $(3x+2)^{\frac{5}{2}} = 4$
 $3x+2 = 32$
 $-2 \quad -2$
 $3x = 30$
 $x = 10$

13. $\sqrt{2x-5} - \sqrt{x-3} = 1$
 $+\sqrt{x-3} \quad +\sqrt{x-3}$
 $(\sqrt{2x-5})^2 = (\sqrt{x-3} + 1)^2$
 $2x-5-1+3 = 1+2\sqrt{x-3}+x-3$
 $x-3 = 2\sqrt{x-3}+x-2$
 $-x+3 \quad -x+3$
 $-2 = 2\sqrt{x-3}$
 $-1 = \sqrt{x-3}$
 $(\sqrt{x-3})^2 = (-1)^2$
 $x-3 = 1$
 $x = 4$ **extraneous**

14. $\sqrt{x+2} + \sqrt{3x+4} = 2$
 $(\sqrt{x+2})^2 + 2\sqrt{x+2}\sqrt{3x+4} + (\sqrt{3x+4})^2 = 2^2$
 $x+2+4+2\sqrt{3x+4}\sqrt{x+2}+3x+4 = 4$
 $4x+10+2\sqrt{3x+4}\sqrt{x+2} = 4$
 $-4x-6 \quad -4x-6$
 $2\sqrt{3x+4}\sqrt{x+2} = -4$
 $\sqrt{3x+4}\sqrt{x+2} = -2$
 $(\sqrt{3x+4}\sqrt{x+2})^2 = (-2)^2$
 $(3x+4)(x+2) = 4$
 $3x^2+10x+8 = 4$
 $3x^2+10x+4 = 0$
 $(3x+4)(x+1) = 0$
 $x = -4/3 \quad x = -1$ **extraneous**