

## MODEL &amp; DISCUSS

In business, the term *profit* is used to describe the difference between the money the business earns (revenue) and the money the business spends (cost).

- A. Grooming USA charges \$25 for every pet that is groomed. Let  $x$  represent the number of pets groomed in a month. Define a revenue function for the business.

$$r(x) = 25x$$



- B. Materials and labor for each pet groomed costs \$15. The business also has fixed costs of \$1,000 each month. Define a cost function for this business.

$$c(x) = 15x + 1000$$

- C. Last month, Grooming USA groomed 95 pets. Did they earn a profit? What would the profit be if the business groomed 110 pets in a month?

$$\begin{aligned} r(95) &= 25(95) = \$2375 \\ c(95) &= 15(95) + 1000 = \$2425 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Loss, } \textcircled{!} \text{ no profit.}$$

$$\begin{aligned} r(110) &= 25(110) = \$2750 \\ c(110) &= 15(110) + 1000 = \$2650 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{profit } \$100$$

- D. **Generalize** Explain your procedure for calculating the profit for Grooming USA. Suppose you wanted to calculate the profit for several different scenarios. How could you simplify your process? © MP.8

$$P(x) = r(x) - c(x)$$

## HABITS OF MIND

**Make Sense and Persevere** A business breaks even when its revenue equals its costs. How many pets would Grooming USA have to groom in order to break even? © MP.1

$$\begin{aligned} P(x) &= r(x) - c(x) \\ 0 &= r(x) - c(x) \\ c(x) &= r(x) \\ 15x + 1000 &= 25x \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{revenue} = \text{cost}$$

$$1000 = 10x$$

$$100 = x$$

pets

Combine like-terms



**EXAMPLE 1** Try It! Add and Subtract Functions

1. Let  $f(x) = 2x^2 + 7x - 1$  and  $g(x) = 3 - 2x$ . Identify rules for the following functions.

a.  $f + g$

$$\begin{aligned} &\rightarrow 2x^2 + 7x - 1 + 3 - 2x \\ &\rightarrow 2x^2 + 5x + 2 \end{aligned}$$

b.  $f - g$

$$\begin{aligned} &\rightarrow (2x^2 + 7x - 1) - (3 - 2x) \\ &\quad 2x^2 + 7x - 1 - 3 + 2x \\ &\rightarrow 2x^2 + 9x - 4 \end{aligned}$$

**EXAMPLE 2** Try It! Multiply Functions

2. Suppose demand,  $d$ , for a company's product at a cost,  $x$ , predicted by the function  $d(x) = -0.25x^2 + 1,000$ , and the price,  $p$ , that the company can charge for the product is given by  $p(x) = x + 16$ . Find the company's revenue function.

demand,  $d(x)$

price,  $p(x)$

revenue,  $r(x)$

$$\begin{aligned} r(x) &= d(x) \cdot p(x) \\ &= (-0.25x^2 + 1000)(x + 16) \end{aligned}$$

$$r(x) = -0.25x^3 - 4x^2 + 1000x + 16000$$

**EXAMPLE 3** Try It! Divide Functions

3. Identify the rule and domain for  $\frac{f}{g}$  for each pair of functions.

a.  $f(x) = x^2 - 3x - 18$ ,  $g(x) = x + 3$

b.  $f(x) = x - 3$ ,  $g(x) = x^2 - x - 6$

$$\frac{f}{g} \rightarrow \frac{(x^2 - 3x - 18)}{(x + 3)} \rightarrow \frac{(x - 6)(x + 3)}{(x + 3)}$$

$$\frac{f}{g} = (x - 6) \quad d: x \neq -3$$

$$\frac{f}{g} \rightarrow \frac{(x - 3)}{(x^2 - x - 6)} \rightarrow \frac{(x - 3)}{(x - 3)(x + 2)}$$

$$\frac{f}{g} \rightarrow \frac{1}{x + 2} \quad d: x \neq 3, -2$$

cannot  $\div 0 \rightarrow$  domain  $g \neq 0$

hole v.a.

**HABITS OF MIND**

**Communicate Precisely** How are the domains of  $f + g$ ,  $f - g$ ,  $f \cdot g$ , and  $\frac{f}{g}$  related to the domains of  $f$  and  $g$ ? © MP.6

$g(x) \neq 0$

Composition of Functions <sup>turkey</sup> "f of g of x" <sup>Asses</sup> a function within another function

Notes  $(f \circ g)(x) = f(g(x))$

"fog"  
4c)  $(f \circ f)(x)$   
→  $2(2x-1)-1$   
→  $4x-2-1$   
→  $4x-3$

EXAMPLE 4 **Try It!** <sup>stuffing</sup> **Compose Functions**

4. Let  $f(x) = 2x - 1$  and  $g(x) = 3x$ . Identify the rules for the following functions.

a.  $f(g(2))$

$g(2) = 3(2)$   
 $g(2) = 6$   
 $f(6) = 2(6) - 1$   
 $f(6) = 12 - 1$   
 $= 11$

b.  $f(g(x))$

$2(3x) - 1$   
 $6x - 1$

OR  $f(g(2)) = 2(3(2)) - 1$   
 $= 12 - 1$   
 $= 11$

EXAMPLE 5 **Try It!** **Write a Rule for a Composite Function**

5. Identify the rules for  $f \circ g$  and  $g \circ f$ .

a.  $f(x) = x^3$ ,  $g(x) = x + 1$

$f \circ g \rightarrow (x+1)^3$  or  $x^3 + 3x^2 + 3x + 1$

$g \circ f \rightarrow (x^3) + 1$

b.  $f(x) = x^2 + 1$ ,  $g(x) = x - 5$

$f \circ g \rightarrow (x-5)^2 + 1$  or  $x^2 - 10x + 26$

$g \circ f \rightarrow (x^2 + 1) - 5 \rightarrow x^2 - 4$

EXAMPLE 6 **Try It!** **Use a Composite Function Model**

6. As a member of the Games Shop rewards program, you get a 20% discount on purchases. All sales are subject to a 6% sales tax. Write functions to model the discount and the sales tax, then identify the rule for the composition function that calculates the final price you would pay at the Games Shop.



**HABITS OF MIND**

**Reason** Let  $f(x) = \frac{1}{3}(x - 2)$  and  $g(x) = 3x + 2$ . Find the rules for  $f \circ g$  and  $g \circ f$ . Are they equivalent? If so, does this prove that the composition of two functions is commutative? **MP2**

$f \circ g = \frac{1}{3}((3x+2)-2)$   
 $f \circ g = \frac{1}{3}(3x)$   
 $= x$

$g \circ f = 3(\frac{1}{3}(x-2)) + 2$   
 $= x - 2 + 2$   
 $= x$

### Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you combine, multiply, divide, and compose functions, and how do you find the domain of the resulting function?

2. **Vocabulary** In your own words, define and provide an example of a composite function.

3. **Error Analysis** Reagan said the domain of  $\frac{f}{g}$  when  $f(x) = 5x^2$  and  $g(x) = x + 3$  is the set of real numbers. Explain why Reagan is incorrect. © MP.3

4. **Make Sense and Persevere** Explain why changing the order in which two functions occur affects the result when subtracting and dividing the functions. © MP.1

### Do You KNOW HOW?

Let  $f(x) = 3x^2 + 5x + 1$  and  $g(x) = 2x - 1$ .

5. Identify the rule for  $f + g$ .

$$\rightarrow 3x^2 + 5x + 1 + 2x - 1$$

$$\rightarrow \boxed{3x^2 + 7x}$$

6. Identify the rule for  $f - g$ .

$$\rightarrow (3x^2 + 5x + 1) - (2x - 1)$$

$$\rightarrow 3x^2 + 5x + 1 - 2x + 1$$

$$\rightarrow \boxed{3x^2 + 3x + 2}$$

7. Identify the rule for  $g - f$ .

$$(2x - 1) - (3x^2 + 5x + 1)$$

$$\rightarrow 2x - 1 - 3x^2 - 5x - 1$$

$$\rightarrow \boxed{-3x^2 - 3x - 2}$$

Let  $f(x) = x^2 + 2x + 1$  and  $g(x) = x - 4$ .

8. Identify the rule for  $f \cdot g$ .

$$(x^2 + 2x + 1) \cdot (x - 4)$$

	$x^2$	$2x$	$+1$	
$x$	$x^3$	$2x^2$	$x$	
$-4$	$-4x^2$	$-8x$	$-4$	

$$\rightarrow \boxed{x^3 - 2x^2 - 7x - 4}$$

9. Identify the rule for  $\frac{f}{g}$ , and state the domain.

$$\frac{x^2 + 2x + 1}{x - 4} \rightarrow \frac{(x+1)(x+1)}{(x-4)}$$

$x - 4 \neq 0$   
domain:  $x \neq 4$

10. Identify the rule for  $\frac{g}{f}$ , and state the domain.

$$\frac{x - 4}{x^2 + 2x + 1} \rightarrow \frac{x - 4}{(x-1)(x-1)} \quad d: x \neq 1$$

11. If  $f(x) = 2x^2 + 5$  and  $g(x) = -3x$ , what is  $f(g(x))$ ?

$(f \circ g)(x)$

$$\rightarrow 2(-3x)^2 + 5$$

$$\rightarrow \boxed{18x^2 + 5}$$