Activity

5-6 Inverse Relations and Functions

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EXPLORE & REASON

Each number path will lead you from a number in the domain, the set of all real numbers, to a number in the range.

PE
MD
AS

Number Path $f: x \to f(x)$
- Start with x.
- Subtract 3.
- Multiply by -2.
- Add 5.

Number Path $g: x \to g(x)$
- Start with x.
- Add 1.
- Square the value.
- Subtract 2.

A. Follow the number paths to find $f(1)$ and $g(1)$.

$f(1) = (1-3)(-2)+5$
$f(1) = 9$

$g(1) = (1+1)^2 - 2$
$g(2) = 2$

B. Identify all possible values of x that lead to $\boxed{f(x) = 7}$ and all values that lead to $\boxed{g(x) = 7.}$

y y y

$f(x) = 7$ • go backwards
$y = 7 - 5 = 2 \div -2 = -1 + 3 = \boxed{2}$ x

$g(x) = 7$
$= 7 + 2 = \pm\sqrt{9}$
$= \pm 3 - 1$
$= \boxed{2}, \boxed{-4}$

two solutions

C. Communicate Precisely Based on the two number paths, under what conditions can you follow a path back to a unique value in the domain? MP.6

HABITS OF MIND

Model With Mathematics Write a rule for Number Path f. Write a rule for the process of following the number path backward. How do the two rules compare? MP.4

$f(x) = (x-3) \cdot -2 + 5$
$f(x) = -2(x-3) + 5$

Path Backwards
$= \dfrac{f(x) - 5}{-2} + 3$

Function → Every x has a unique y-value
• also vert line test

Notes

Swapping domain & range

EXAMPLE 1 Try It! Represent the Inverse of a Relation

1. Identify the inverse relation. Is it a function?

x	−1	0	1	2	3	4
y	9	7	5	3	1	−1

x	9	7	5	3	1	−1
y	−1	0	1	2	3	4

x → y : true
→ the inverse is a function..

To find an inverse:
• Swap x & y
• Isolate y....

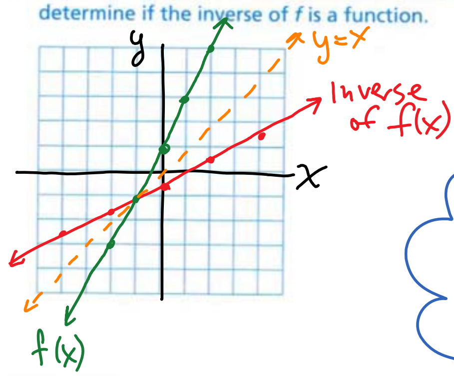
EXAMPLE 2 Try It! Find an Equation of an Inverse Relation

2. Let $f(x) = 2x + 1$.

a. Write an equation to represent the inverse of f.

$x = 2y + 1$
$-1 \quad -1$
$\frac{x-1}{2} = \frac{2y}{2}$
$\frac{x-1}{2} = y$ or $\boxed{y = \frac{1}{2}x - \frac{1}{2}}$

b. Sketch the graphs of f and its inverse on the same coordinate axes and determine if the inverse of f is a function.



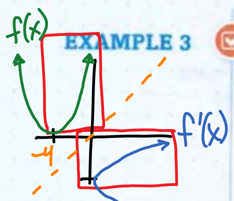
aka $f^{-1}(x)$
** • is not necessarily a function
** • $f^{-1}(x) \neq \frac{1}{f(x)}$

HABITS OF MIND

Communicate Precisely Think of $f(x) = 2x + 1$ as a number path: start with x, multiply by 2, and add 1. How could you describe the path from the result back to x? MP.6

• subt 1
• divide by 2

EXAMPLE 3 Try It! Restrict a Domain to Produce an Inverse Function



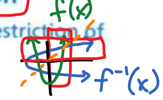
$f(x) = x^2 + 8x + 16$
d: $x \geq -4$
restriction

3. Find the inverse of each function by identifying an appropriate restriction of its domain.

a. $f(x) = x^2 + 8x + 16$ parabola

$x = y^2 + 8y + 16$ PST
$x = (y+4)(y+4)$
$x = (y+4)^2$
$\pm\sqrt{x} \quad \sqrt{(y+4)^2}$
$\pm\sqrt{x} = y + 4$
$-4 \pm \sqrt{x} = y$ not a function
$\boxed{-4 + \sqrt{x} = y}$ "top half"

b. $f(x) = x^2 - 9$ parabola
d: $x \geq 0$
$x = y^2 - 9$
$x + 9 = y^2$
$\pm\sqrt{x+9} = \sqrt{y^2}$
$\pm\sqrt{x+9} = y$
not a function
$\sqrt{x+9} = y$ "top half"



Notes

Verify

If $(f \circ g)(x) = (g \circ f)(x) = x$, then f & g are inverses of one another...

Assess

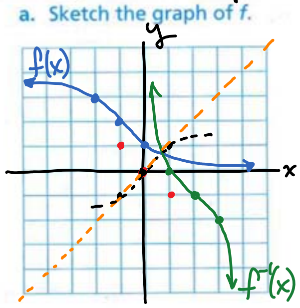
$a\sqrt[3]{x-h}+k$

refl $a=-1$ up k h: left

EXAMPLE 4 Try It! Find an Equation of an Inverse Function

4. Let $f(x) = 2 - \sqrt[3]{x+1}$.

a. Sketch the graph of f.



b. Verify that the inverse will be a function and write an equation for $f^{-1}(x)$.

$x = 2 - \sqrt[3]{y+1}$

$x - 2 = -\sqrt[3]{y+1}$

$-x+2 = \sqrt[3]{y+1}$

$(-x+2)^3 = y+1$

$\boxed{(-x+2)^3 - 1 = y}$

$(f \circ f^{-1})(x) = 2 - \sqrt[3]{((-x+2)^3 - 1) + 1} \to 2 - \sqrt[3]{(-x+2)^3} \to 2 - (-x+2) \to x$

$(f^{-1} \circ f)(x) = (-(2 - \sqrt[3]{x+1}) + 2)^3 - 1 \to (-2 + \sqrt[3]{x+1} + 2)^3 - 1 \to (\sqrt[3]{x+1})^3 - 1 \to x + 1 - 1 \to x$

EXAMPLE 5 Try It! Use Composition to Verify Inverse Functions

5. Use composition to determine whether f and g are inverse functions.

a. $f(x) = \frac{1}{4}x + 7$, $g(x) = 4x - 7$

$f(g(x)) = \frac{1}{4}(4x-7)+7$

$= x - \cancel{\frac{7}{4}} + 7$!!

~~not~~ inverses...

b. $f(x) = \sqrt[3]{x-1}$, $g(x) = x^3 + 1$

$f(g(x)) = \sqrt[3]{(x^3+1)-1} = \sqrt[3]{x^3} = x$

$g(f(x)) = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$

f(x) & g(x) are inverses!

HABITS OF MIND

Construct Arguments Dana says that the functions $f(x) = (x-2)^2 + 5$ and $g(x) = \sqrt{x-5} + 2$ are inverses. Keegan says that the functions are inverses only if the domain is restricted. Is either person correct? Explain. MP.3

EXAMPLE 6 Try It! Rewrite a Formula

6. The manufacturer of a gift box designs a box with length and width each twice as long as its height. Find a formula that gives the height h of the box in terms of its volume V. Then give the length of the box if the volume is 640 cm^3.

HABITS OF MIND

Make Sense and Persevere In the formula $V = \frac{4}{3}\pi r^3$, which variable is the dependent variable? In the formula $r = \sqrt[3]{\frac{3}{4\pi}V}$, which variable is the dependent variable? MP.1

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you find the inverse of a function and verify the two functions are inverses?

2. **Error Analysis** Abi said the inverse of $f(x) = 3x + 1$ is $f^{-1}(x) = \frac{1}{3}x - 1$. Is she correct? Explain. MP.3

3. **Construct Arguments** Is the inverse of a function always a function? Explain. MP.3

Do You KNOW HOW?

Consider the function $f(x) = -\frac{1}{2}x + 5$.

4. Write an equation for the inverse of $f(x)$.

$x = -\frac{1}{2}y + 5$

$x - 5 = -\frac{1}{2}y$

$-2(x-5) = y$

$-2x + 10 = y$

$-2x + 10 = f^{-1}(x)$

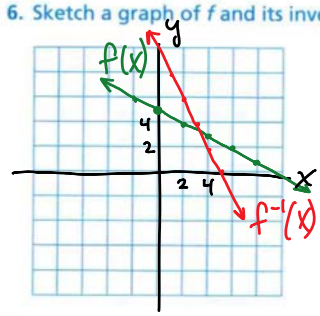
5. Use composition to determine whether the inverse of $f(x)$ is a function.

$f(f^{-1}(x)) = -\frac{1}{2}(-2(x-5)) + 5 = (x-5)+5 = x$

$f^{-1}(f(x)) = -2((-\frac{1}{2}x+5)-5)$

$= -2(-\frac{1}{2}x) = x$

6. Sketch a graph of f and its inverse.



$f(x) = -\frac{1}{2}x + 5$

$f^{-1}(x) = -2x + 10$

7. How can you verify by the graph of f and its inverse that they are indeed inverses?

$f(f^{-1}(x)) = -\frac{1}{2}(-2x+10) + 5 = x - 5 + 5 = x$

$f^{-1}(f(x)) = -2(-\frac{1}{2}x+5) + 10 = x - 10 + 10 = x$

8. Is the inverse of $f(x)$ a function? Explain.

Yes... passes vertical line test