Activity

6-1 Key Features of Exponential Functions

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EXPLORE & REASON

Margaret investigates three functions: $y = 3x$, $y = x^3$, and $y = 3^x$. She is interested in the differences and ratios between consecutive y-values. Here is the table she started for $y = 3x$.

linear

Investigating $y = 3x$			
x	y	Difference between y-values	Ratio between y-values
1	3		
2	6	$6 - 3 = 3$	$\frac{6}{3} = 2$
3	9	$9 - 6 = 3$	$\frac{9}{6} = 1.5$
4	12	$12 - 9 = 3$	$\frac{12}{9} \approx 1.33$

current − previous

Common difference: 3

no common ratio

A. Create tables like Margaret's for all three functions and fill in more rows.

linear: $y = 3x$; cubic: $y = x^3$; exponential growth: $y = 3^x$

x	$y = 3x$	$y = x^3$	$y = 3^x$
1	3	1	3
2	6	8	9
3	9	27	27
4	12	64	81

(×8, ×?, ×?) (×3, ×3, ×3)

B. Which functions have a constant difference between consecutive y-values? Constant ratio?

$y = 3x$ (linear)

$r: 3$ ← 300%.

$y = 3^x$ (exponential)

C. Reason Which of these three functions will have y-values that increase the fastest as x increases? Why? MP.2

→ exponential growth

- look at the graphs...

x^3 3^x

x^4 4^x

x^5 5^x

x^6 6^x ← 600% faster

HABITS OF MIND

Generalize Let b represent a whole number. For $b > 1$, which function do you think will increase at a faster rate as x increases, $f(x) = b^x$ or $g(x) = x^b$? Explain. MP.8

exponential growth / decay

b^x vs x^b

b ← base

x^b ← base; polynomial

→ faster...

$f(x) = a \cdot b^{x-h} + k$

$f(x) = a \cdot b^{x-h} + k$

Notes

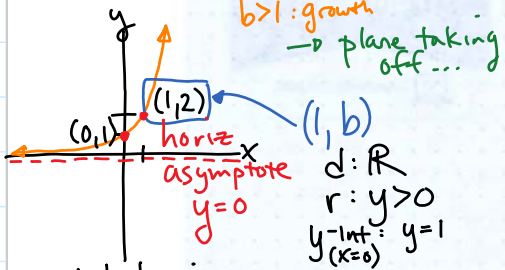
Assess

ex) $f(x) = 2^x$

x	y
0	1
1	2

a: 1 b: 2

b > 1 : growth → plane taking off...



d: ℝ
r: $y > 0$
$y\text{-int}_{(x=0)}$: $y = 1$

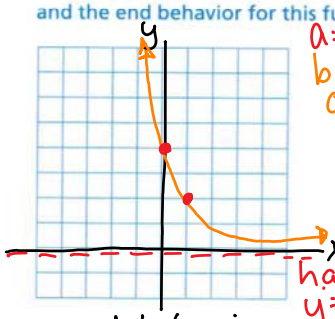
end behavior (from the middle)
As $x \to -\infty$ (left), $y \to 0$ (x-axis)
As $x \to +\infty$ (right), $y \to \infty$ (up)

EXAMPLE 1 **Try It! Identify Key Features of Exponential Functions**

1. Graph $f(x) = 4(0.5)^x$. What are the domain, range, intercept, asymptote, and the end behavior for this function?

a: 4 vert stretch
b: 0.5
$0 < b < 1$ decay

x	y
0	4
1	2



d: ℝ
r: $y > 0$
$y\text{-int}_{(x=0)}$: $y = 4$

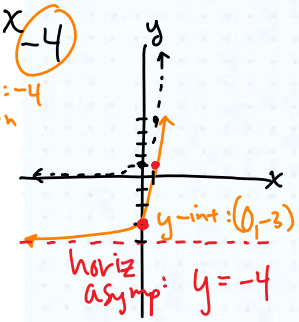
end behavior
As $x \to -\infty$, $y \to \infty$
As $x \to +\infty$, $y \to 0$

$a \cdot b^{x-h} + k$

EXAMPLE 2 **Try It! Graph Transformations of Exponential Functions**

c) $5^x - 4$

k: −4 down

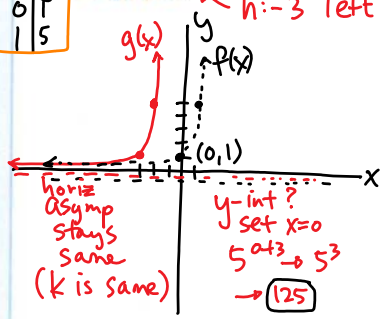


parent 5^x

x	y
0	1
1	5

2. How do the asymptote and intercept of the given function compare to the asymptote and intercept of the function $f(x) = 5^x$?

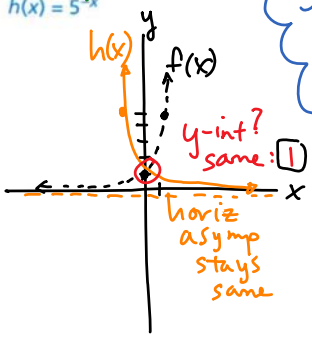
a. $g(x) = 5^{x+3}$ h: −3 left



b. $h(x) = 5^{-x}$ growth multiplier

horiz refl or

$5^{-x} \to \frac{1}{5^x} \to \left(\frac{1}{5}\right)^x$ decay



HABITS OF MIND

Reason What kinds of transformations will affect the asymptote or the intercept(s) of an exponential function? Explain. MP.2

a, h, k

vert shifts: k

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$y = a \cdot b^x$ ⇨ $a(1+r)^t$ or $a(1-r)^t$ (initial (when $x/t=0$); rate; growth; decay; rate)

base? growth/decay

Notes

Assess

EXAMPLE 3 **Try It! Model with Exponential Functions**

3. A factory purchased a 3D Printer in 2010. The value of the printer is modeled by the function $f(x) = 30(0.93)^x$ where x is the number of years since 2010.

a. What is the value of the printer after 10 years?

$f(10) = 30(0.93)^{10}$
$= 14.5$

b. Does the printer lose more of its value in the first 10 years or in the second?

$f(20) = 30(0.93)^{20}$
$= 7.0$

2010 → 2020: $30 - 14.5 = 15.5$
2020 → 2030: $14.5 - 7.0 = 7.5$

Lost more the 1st 10 yrs...

EXAMPLE 4 **Try It! Interpret an Exponential Function**

4. Two-hundred twenty hawks were released into a region in 2016. The function $f(x) = 220(1.05)^x$ can be used to model the number of red-tailed hawks in the region x years after 2016.

a. Is the population increasing or decreasing? Explain.

1.05
1 + .05 or 100% + 5%

5% growth: increasing

b. In what year will the number of hawks reach 280? ($f(x)$)

$280 = 220(1.05)^x$?

Trace GC... $x \approx 5$ years or 2021

HABITS OF MIND

Use Structure How can you determine the growth or decay factor by looking at an exponential function? The growth or decay rate? MP.7

from the textbook...

Sculpture
$f(x) = 50(1.075)^x$

painting value

(5, 64.4)

(0, 40)

$ 1000's

In 5 years, which piece has a higher rate of change? → Slope

EXAMPLE 5 **Try It! Compare Two Exponential Functions**

5. In Example 5, will the value of the painting ever surpass the value of the sculpture according to the models? Explain.

..... maybe

HABITS OF MIND

Reason For two functions $f(x) = b^x$ and $g(x) = b^{x+n}$ where $n > 0$, is it possible that the two graphs will intersect? Explain. MP.2

h: negative → left

b^{x+n} b^x

point of intersection? NO

... $f(x) = g(x)$
$b^x = b^{x+n}$
$x = x + h$
$-x \quad -x$
$0 = h$ False

sculpture
$f(0) = 50(1.075)^0 = 50$
$f(5) = 50(1.075)^5 \approx 71.78$
slope $= \frac{71.78 - 50}{5 - 0} = \frac{21.78}{5} \approx 4.356$

painting
slope $= \frac{64.4 - 40}{5 - 0} = \frac{24.4}{5} \approx 4.88$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do graphs and equations reveal key features of exponential growth and decay functions?

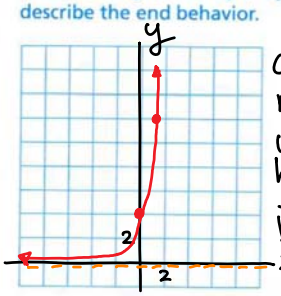
2. **Vocabulary** How do *exponential functions* differ from polynomial and rational functions?

3. **Error Analysis** Charles claimed the function $f(x) = \left(\frac{3}{2}\right)^x$ represents exponential decay. Explain the error Charles made. MP.3

4. **Communicate Precisely** How are exponential growth functions similar to exponential decay functions? How are they different? MP.6

Do You KNOW HOW?

5. Graph the function $f(x) = 4 \cdot 3^x$. Identify the domain, range, intercept, asymptote, and describe the end behavior.



d: $\mathbb{R}$
r: $y > 0$
y-int: 4
H.a: $y = 0$
end behavior: As $x \to -\infty$, $f(x) \to 0$
As $x \to +\infty$, $f(x) \to \infty$

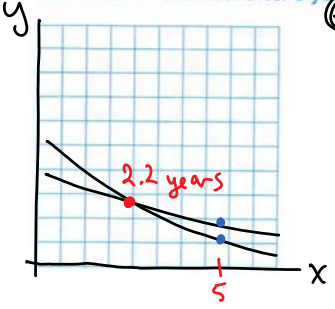
6. The exponential function $f(x) = 2500(0.4)^x$ models the amount of money in Zachary's savings account over the last 10 years. Is Zachary's account balance increasing or decreasing? Write the base in terms of the rate of growth or decay.

decreasing...
base: (0.4) or 40% growth
or 60% decay

7. Describe how the graph of $g(x) = 4(0.5)^{x-3}$ compares to the graph of $f(x) = 4(0.5)^x$.

→ shift right 3

8. Two trucks were purchased by a landscaping company in 2016. Their values are modeled by the functions $f(x) = 35(0.85)^x$ and $g(x) = 46(0.75)^x$ where x is the number of years since 2016. Which function models the truck that is worth the most after 5 years? Explain.



GC or Subst

$f(5) = 35(0.85)^5 = 15.5$ ← worth more.

$g(5) = 46(0.75)^5 = 10.9$

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