Activity

6-2 Exponential Models

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EXPLORE & REASON

Juan is studying exponential growth of bacteria cultures. Each is carefully controlled to maintain a specific growth rate. Copy and complete the table to find the number of bacteria cells in each culture.

Culture	Initial Number of Bacteria	Growth Rate per Day	Time (days)	Final Number of Bacteria
A	10,000	8%	1	10,800
B	10,000	4%	2	10,816
C	10,000	2%	4	10, 824
D	10,000	1%	8	10, 829

or $10000(1.08)$

$\overbrace{10000(.08)+10000}$ or

$(\overbrace{10600(.04)+10000})(1.04)$

$10600(1.02)^4$ $\quad$ $10000(1.04)^2$

$10006(1.01)^8$

A. What is the relationship between the daily growth rate and the time in days for each culture?

B. **Look for Relationships** Would you expect a culture with a growth rate of $\frac{1}{2}\%$ and a time of 16 days to have more or fewer cells than the others in the table? Explain. MP.7

HABITS OF MIND

Construct Arguments Oscar thinks that any function of the form $y = b^x$ has to approach infinity as x approaches infinity. How could you explain to Oscar why that does not happen with the function $y = \left(1+\frac{1}{x}\right)^x$? MP.3

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

r: rate

nt: time (years)

A: Final Amount

P: principal (initial value)

1: 100%

n: # times interest is compounded per year

EXAMPLE 1 **Try It!** Rewrite an Exponential Function to Identify a Rate

growth

4 times

.018

1. The population in a small town is increasing annually at 1.8%. What is the quarterly rate of population increase?

$$y = P(1 + .018)^t = P(1.018)^t$$
$$= P(1.018)^{\frac{4}{4}t} = P\left(\boxed{1.018^{\frac{1}{4}}}\right)^{4t}$$

$1.018 \wedge (1 \div 4)$

$$= P(1.00446994)^{4t}$$

→ about 0.446% per quarter growth

HABITS OF MIND

Generalize Why can't you just divide an annual interest rate by 4 to obtain a quarterly interest rate? MP.8

→ exponential growth... not linear

EXAMPLE 2 **Try It!** Understand Continuously Compounded Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

- monthly: 12
- weekly: 52
- daily: 365
- quarterly: 4
- annually: 1
- bimonthly: 6
- Semimonthly: 24
- ⋮

2. \$3,000 is invested in an account that earns 3% annual interest, compounded monthly.

P

n: 12

r: 0.03

a. What is the value of the account after 10 years?

t

$$A = 3000\left(1 + \frac{.03}{12}\right)^{12 \cdot 10}$$
$$A = \$4048.06$$

b. What is the value of the account after 100 years?

$$A = 3000\left(1 + \frac{.03}{12}\right)^{12(100)}$$
$$= \$60{,}031.46$$

EXAMPLE 3 **Try It!** Understanding Continuously Compounded Interest

3. If you continued the table for n = 1,000,000, would the value in the account increase or decrease? How do you know?

What if n = BIG?

GC

$$A = P\left(1 + \frac{1}{BIG}\right)^{BIG \cdot t}$$

→ approaches...

2.718281828459...

natural base

HABITS OF MIND

Generalize Which yields the greatest return on investment: compounding quarterly, hourly, or continuously? Explain. MP.8

n: 4

n: 8760

e: natural base

→ grow naturally in OUR universe...

→ irrational...

$$3000e^{.03(100)} = \boxed{60256.61}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \Rightarrow \boxed{A = Pe^{rt}}$$

Notes

$e \wedge 3 \stackrel{vs}{=} e3$

Assess

EXAMPLE 4 **Try It!** **Find Continuously Compounded Interest**

$A = Pe^{rt}$

4. You invest $125,000 in an account that earns 4.75% annual interest, compounded continuously.

a. What is the value of the account after 15 years?

$= 125000e^{.0475(15)}$

$= \boxed{254885.32}$

b. What is the value of the account after 30 years?

$= 125000e^{.0475(30)}$

$= \boxed{519732.23}$

EXAMPLE 5 **Try It!** **Use Two Points to Find an Exponential Model**

5. A surveyor determined the value of an area of land over a period of several years since 1950. The land was worth $31,000 in 1954 and $35,000 in 1955. Use the data to determine an exponential model that describes the value of the land.

year 5

year 0

b: growth factor

→ ratio of y-values

"slope"

$y = a \cdot b^x$?

$\frac{35000}{31000} \approx \boxed{\frac{35}{31}}$

(x, y)

$(5, 35000)$

$y = a \cdot \left(\frac{35}{31}\right)^x$

$35000 = a\left(\frac{35}{31}\right)^5$

$19078.15 = a$

$\therefore \boxed{y = 19078.15\left(\frac{35}{31}\right)^x}$

EXAMPLE 6 **Try It!** **Use Regression to Find an Exponential Model**

6. In Example 6, what was the temperature of the soup when the cooling began? According to the model, what was the approximate temperature 35 minutes after cooling started?

$\boxed{\text{Exp Reg}}$

see p309 EX 6)...

GC → STATS → x: time & y: temp

→ graph → CALC → $a \cdot b^x$

→ $\boxed{y = 192.26(0.97)^x}$

HABITS OF MIND

Generalize How can a graph help you determine whether an exponential model is appropriate for a data set? Explain. MP.8

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How can you develop exponential models to represent and interpret situations?

2. **Error Analysis** The exponential model $y = 5{,}000(1.05)^t$ represents the amount Yori earns in an account after t years when \$5,000 is invested. Yori said the monthly interest rate of the exponential model is 5%. Explain Yori's error. MP.3

3. **Vocabulary** Explain the similarities and differences between compound interest and continuously compounded interest. MP.6

4. **Make Sense and Persevere** Kylee is using a calculator to find an exponential regression model. How would you explain to Kylee what the variables in the model $y = ab^x$ represent? MP.1

Do You KNOW HOW?

The exponential function models the annual rate of increase. Find the monthly and quarterly rates.

5. $f(t) = 2{,}000(1.03)^t$

$2000(1.03)^{\frac{12}{12}t}$

$2000(1.03^{\frac{1}{12}})^{12t}$

$2000(\underbrace{1.00246627}_{})^{12t}$

0.246627%

$2000(1.03)^{\frac{4}{4}t}$

$2000(1.03^{\frac{1}{4}})^{4t}$

$2000(1.00741707\!2)^{4t}$

0.7417072%

6. $f(t) = 500(1.055)^t$

$500(1.055)^{\frac{12}{12}t}$

$500(1.055^{\frac{1}{12}})^{12t}$

$500(1.004471699)^{12t}$

0.4471699%

$500(1.055)^{\frac{4}{4}t}$

$500(1.055^{\frac{1}{4}})^{4t}$

$500(1.013475174)^{4t}$

1.3475174%

Find the ~~total~~ Final amount of money ~~invested~~ in an account at the end of the given time period.

7. compounded monthly, $P = \$2{,}000$, $r = 3\%$, $t = 5$ years

$A = P(1+\frac{r}{n})^{nt}$

$= 2000(1+\frac{.03}{12})^{12(5)}$

$= \boxed{\$2323.23}$

8. continuously compounded, $P = \$1{,}500$, $r = 1.5\%$, $t = 6$ years

$A = Pe^{rt}$

$= 1500e^{0.015(6)}$

$= \boxed{\$1641.26}$

Write an exponential model given two points.

9. (3, 55) and $\boxed{(4, 70)}$ x y

$y = a\cdot b^x \rightarrow 70 = a\cdot(\frac{14}{11})^4$

$\frac{70}{55} = \frac{14}{11}$ $\quad = a$

$26.6782607 = a$

$\therefore \boxed{y = 26.7(\frac{14}{11})^x}$

10. (7, 12) and $\boxed{(8, 25)}$ x y

$y = a\cdot b^x \rightarrow 25 = a(\frac{25}{12})^8$

$\frac{25}{12}$ $\quad 0.07044 = a$

$\therefore \boxed{y = 0.07(\frac{25}{12})^x}$

11. Paul invests \$6,450 in an account that earns continuously compounded interest at an annual rate of 2.8%. What is the value of the account after 8 years?

$A = Pe^{rt}$

$= 6450e^{.028(8)}$

$= \$8069.41$