Activity

6-3 Logarithms

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CRITIQUE & EXPLAIN

Earthquakes make seismic waves through the ground. The equation $y = 10^x$ relates the height, or amplitude, in microns, of a seismic wave, y, and the power, or magnitude, x, of the ground-shaking it can cause.

Taylor and Chen used different methods to find the magnitude of the earthquake with amplitude 5,500.

Magnitude, x	Amplitude, y
2	100
3	1,000
?	5,500
4	10,000

Taylor

5,500 is halfway between 1,000 and 10,000.

3.5 is halfway between 3 and 4.

The magnitude is about 3.5.

Chen

$y = 10^x$

$10^3 = 1{,}000$
$10^4 = 10{,}000$
$10^{3.5} \approx 3{,}162$
$10^{3.7} \approx 5{,}012$
$10^{3.8} \approx 6{,}310$
$10^{3.74} \approx 5{,}500$

The magnitude is about 3.74.

A. What is the magnitude of an earthquake with amplitude 100,000? How do you know?

$10^? = 100000$

$? = 5$

B. **Construct Arguments** Critique Taylor's and Chen's work. Is each method valid? Could either method be improved? MP.3

Chen: higher / lower... → interpolation (...guess)

C. Describe how to express the exact value of the desired magnitude.

HABITS OF MIND

Reason Taylor reasoned that since 5,500 was halfway between 1,000 and 10,000, that the magnitude had to be halfway between 3 and 4. What is incorrect about Taylor's reasoning? MP.2

ex) $2^x = 8$ — power: x; argument: 8; base: 2

exponentiation

Logarithms

$\log_b x = y$ iff $b^y = x$

$\log_2 8 = x$ "log base b of x"

$y = \log_b x$ is the inverse of $y = b^x$

Notes

Assess

EXAMPLE 1 **Try It! Understand Logarithms**

1. Write the inverse of $y = 8^x$

swap x & y → $x = 8^y$ (exponential form) → isolate y → convert to LOG form → $\log_8 x = y$

EXAMPLE 2 **Try It! Convert Between Exponential and Logarithmic Forms**

2. a. What is the logarithmic form of $7^3 = 343$? exp form

$\log_7 343 = 3$

b. What is the exponential form of $\log_4 16 = 2$? log form

$4^2 = 16$

Common logarithms

- base 10

"log x" → 10

natural logarithms

- base e

"ln x" → $\log_e x$ or $\ln_e x$

HABITS OF MIND

Communicate Precisely Write a sentence to describe what the equation $\log_a b = c$ means. MP.6

EXAMPLE 3 **Try It! Evaluate Logarithms**

3. What is the value of each logarithmic expression?

a. $\log_3 \left(\frac{1}{81}\right) = x$ → log → exp form

$3^x = \frac{1}{81}$

$3^x = \frac{1}{3^4}$

$3^x = 3^{-4}$

$x = -4$

b. $\log_7 (-7) = x$

$7^x = -7$

HUH?

no solution or undefined

c. $\log_5 5^9 = x$

$5^x = 5^9$

$x = 9$

$\log_b b^x = x$ same bases

$b^{\log_b x} = x$

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Notes

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EXAMPLE 4 **Try It! Evaluate Common and Natural Logarithms**

4. What is the value of each logarithmic expression to the nearest ten-thousandth?

GC

a. $\log 321$

≈ 2.5065

$10^{2.5065} \approx 321$

b. $\ln 1{,}215$

≈ 7.1025

$e^{7.1025} \approx 1215$

c. $\log 0.17$

≈ -0.7696

d. $\ln(-1.87)$

$\approx$ error

undefined

HABITS OF MIND

Reason In order for $\log x$ or $\ln x$ to be defined, what must be true about x? MP.2

$x > 0$ positive...

EXAMPLE 5 **Try It! Solve Equations With Logarithms**

$\ln e^x = x$

$e^{\ln x} = x$

5. Solve each equation. Round to the nearest thousandth.

log → exp

a. $\log(3x - 2) = 2$

$10^2 = 3x - 2$

$100 = 3x - 2$

$+2 \quad +2$

$102 = 3x$

$\frac{102}{3} = \frac{3x}{3}$

$34 = x$

b. $e^{x+2} = 8$

$\ln_e 8 = x + 2$

$-2 \quad -2$

$\ln 8 - 2 = x$

$0.079 \approx x$

EXAMPLE 6 **Try It! Use Logarithms to Solve Equations**

6. What is the magnitude of an earthquake with a seismic energy of 1.8×10^{23} joules?

$x = 10^{1.5m + 12}$

exp → log

$1.8 \times 10^{23} = 10^{1.5m+12}$

$\log_{10} 1.8 \times 10^{23} = 1.5m + 12$

$\frac{\log 1.8 \times 10^{23} - 12}{1.5} = m$

$7.504 = m$

HABITS OF MIND

Make Sense and Persevere How do logarithms help you to solve an equation in which the variable is an exponent? MP.1

→ relocates the variable so it's solvable!

Assess

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What are logarithms and how are they evaluated?

2. **Error Analysis** Amir said the expression $\log_5(-25)$ simplifies to -2. Explain Amir's possible error. MP.3

3. **Vocabulary** Explain the difference between the common logarithm and the natural logarithm.

4. **Make Sense and Persevere** How can logarithms help to solve an equation such as $10^t = 656$? MP.1

Do You KNOW HOW?

Write each equation in logarithmic form.

5. $2^{-6} = \frac{1}{64}$ → $\log_2\left(\frac{1}{64}\right) = -6$

6. $e^4 \approx 54.6$ → $\ln 54.6 \approx 4$

Write each equation in exponential form.

7. $\log 200 \approx 2.301$ → $10^{2.301} \approx 200$

8. $\ln 25 \approx 3.22$ → $e^{3.22} \approx 25$

Evaluate the expression.

9. $\log_4 64 = x$ → $4^x = 64 \rightarrow 4^x = 4^3 \rightarrow x = 3$

10. $\log_{10} \frac{1}{100} = x$ → $10^x = \frac{1}{100} \rightarrow 10^x = \frac{1}{10^2} \rightarrow 10^x = 10^{-2}$, $x = -2$

11. $\ln e^5 = x$ → $e^x = e^5 \rightarrow x = 5$

12. Solve for x. $4e^x = 7$.

$e^x = \frac{7}{4} \rightarrow \ln \frac{7}{4} = x$

$0.5596 \approx x$

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