

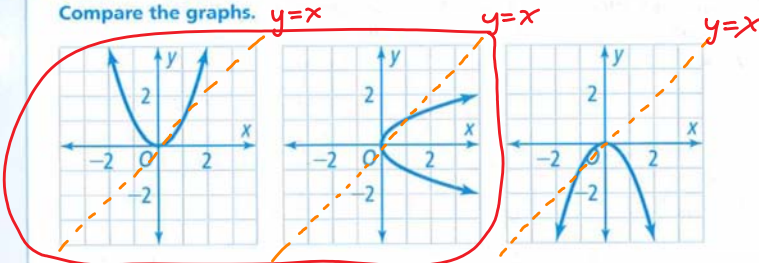
6-4

Logarithmic Functions

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EXPLORE & REASON

Compare the graphs. $y=x$



A. Which two graphs represent the inverse of each other? Explain.

- Swap x & y
- mirror image $y=x$

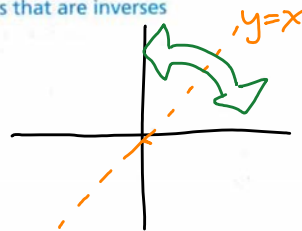
B. **Look for Relationships** What is the relationship between the domain and the range of the two inverse relations? © MP.7

- domain & range are swapped...

HABITS OF MIND

Communicate Precisely How are the points on graphs of functions that are inverses of each other related? © MP.6

- mirror imaged...



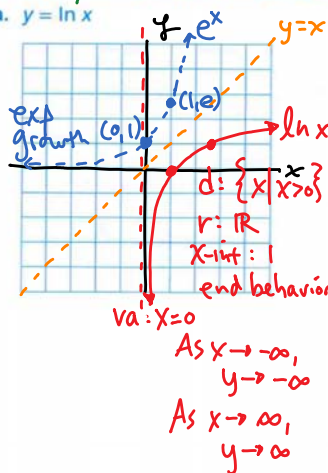
$$\log_b x = y \text{ iff } b^y = x$$

EXAMPLE 1

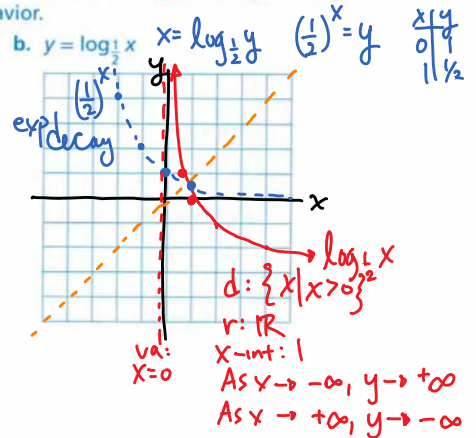
Try It! Identify Key Features of Logarithmic Functions

1. Graph each function and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.

a. $y = \ln x$



b. $y = \log_{1/2} x$



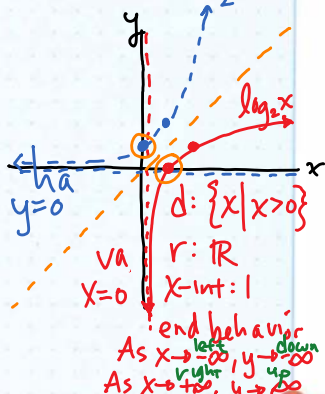
→ Inverse ...

1) $y = \log_2 x$ $b^x = y$

Find the inverse ...

$x = \log_2 y$

$2^x = y$
 $d: \mathbb{R}$
 $r: \{y | y > 0\}$
 $y\text{-int}: 1$

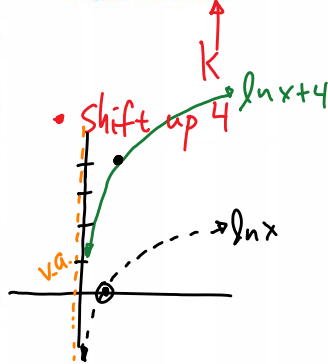


EXAMPLE 2

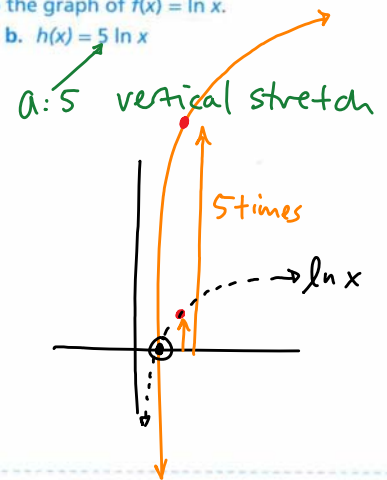
Try It! Graph Transformations of Logarithmic Functions

2. Describe how each graph compares to the graph of $f(x) = \ln x$.

a. $g(x) = \ln x + 4$



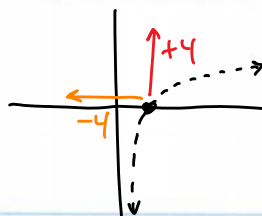
b. $h(x) = 5 \ln x$



a. $\ln(x-h) + k$

HABITS OF MIND

Use Structure Does the graph of either $y = \ln x + 4$ or $y = \ln(x + 4)$ have an intercept that is different from the intercept of $y = \ln x$? Explain. MP.7



k: 4
up 4

h: -4
left 4

EXAMPLE 3 Try It! Inverses of Exponential and Logarithmic Functions

3. Find the inverse of each function.

a. $f(x) = 3^{x+2}$
 $y = 3^{x+2}$
 $x = 3^{y+2}$ exp \rightarrow log.

$\log_3 x = y+2$ or
 $\log_3 x - 2 = y$ $f^{-1}(x) = \log_3 x - 2$

b. $g(x) = \log_7 x - 2$
 $y = \log_7 x - 2$
 $x = \log_7 y - 2$

$x+2 = \log_7 y$
 $7^{x+2} = y$ $f^{-1}(x) = 7^{x+2}$

• Swap x & y ... then isolate y.

EXAMPLE 4 Try It! Interpret the Inverse of a Formula Involving Logarithms

4. Describe what happens to the amount of monthly revenue as the amount of advertising increases. How might you determine the optimal advertising budget? Explain.

$R = 12 \log(a+1) + 25$
 isolate the log 1st...
 $R - 25 = 12 \log(a+1)$
 $\frac{R-25}{12} = \log(a+1)$
 $10^{\frac{R-25}{12}} = a+1$
 $10^{\frac{R-25}{12}} - 1 = a$

HABITS OF MIND

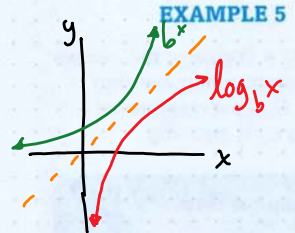
Generalize How would you explain, in your own words, how to find the inverse of a logarithmic function? **MP.8**

isolate the log function before swapping x & y (& isolating y)....



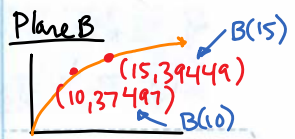
EXAMPLE 5 Try It! Compare Two Logarithmic Functions

5. For which plane do you think the altitude will change more quickly over the interval $10 \leq t \leq 15$? Explain your reasoning.



Plane A
 $A = 9200 \ln t + 10000$

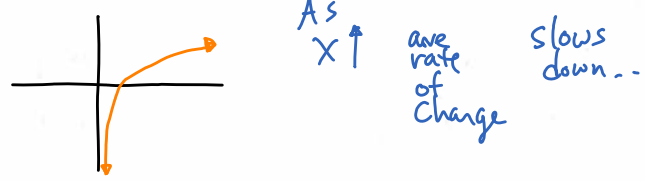
$A(10) = 9200 \ln 10 + 10000 \approx 31184$
 $A(15) = 9200 \ln 15 + 10000 \approx 34914$



average rate of change $\frac{34914 - 31184}{15 - 10} \approx 746$ ft./min
 $\frac{39449 - 37497}{15 - 10} \approx 390$ ft./min

HABITS OF MIND

Look for Relationships How does the average rate of change of the function $f(x) = \log x$ change as x increases? **MP.7**



Do You UNDERSTAND?

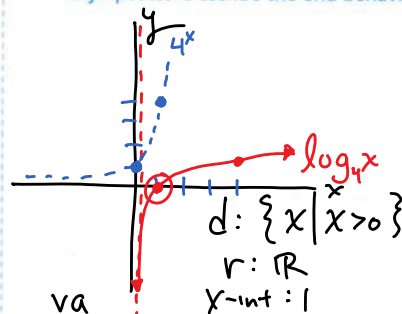
1. **ESSENTIAL QUESTION** How is the relationship between logarithmic and exponential functions revealed in the key features of their graphs?

2. **Error Analysis** Raynard claims the domain of the function $y = \log_3 x$ is all real numbers. Explain the error Raynard made. © MP.3

3. **Communicate Precisely** How are the graphs of $f(x) = \log_5 x$ and $g(x) = -\log_5 x$ related? © MP.6

Do You KNOW HOW?

4. Graph the function $y = \log_4 x$ and identify the domain and range. List any intercepts or asymptotes. Describe the end behavior.



As $x \rightarrow -\infty, y \rightarrow -\infty$
As $x \rightarrow +\infty, y \rightarrow +\infty$

5. Write the equation for the function $g(x)$, which can be described as a vertical shift $1\frac{1}{2}$ units up from the function $f(x) = \ln x - 1$.

$a \ln(x-h) + k$
k: $1\frac{1}{2}$

$g(x) = f(x) + k = \ln x - 1 + k$
 $g(x) = f(x) + 1\frac{1}{2} = \ln x - 1 + 1\frac{1}{2}$

$g(x) = \ln x + \frac{1}{2}$

6. The function $y = 5 \ln(x + 1)$ gives y , the number of downloads, in hundreds, x minutes after the release of a song. Find the equation of the inverse and interpret its meaning.



$\frac{x}{5} = \frac{5 \ln(y+1)}{5}$

$\frac{x}{5} = \ln(y+1)$ log \rightarrow exp

$e^{\frac{x}{5}} = y+1$

$e^{\frac{x}{5}} - 1 = y$