Activity

6-5 Properties of Logarithms

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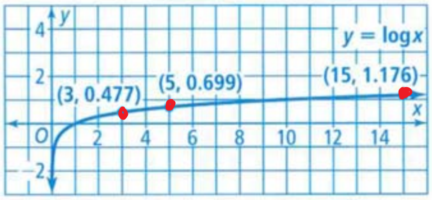
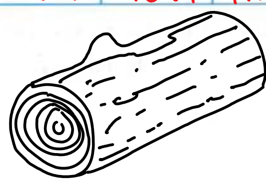
EXPLORE & REASON

Look at the graph of $y = \log_{10} x$ and the ordered pairs shown.



A. Complete the table shown.

x	3	5	15
$\log x$	0.477	0.699	1.176



B. **Look for Relationships** What is the relationship between the numbers 3, 5, and 15? What is the relationship between the logarithms of 3, 5, and 15? MP.7

$3 \cdot 5 = 15$

$\log 3 + \log 5 = \log 15$

$0.477 + 0.699 = 1.176$

C. What is your prediction for the value of log 45? log 75? Explain.

$\log 3 + \log 15$ → $\log 5 + \log 15$

HABITS OF MIND

Generalize Do you think that the relationships you found in the Explore & Reason activity would also hold for natural logarithms? Give an example. MP.8

Is $\ln 3 + \ln 15 = \ln 45$?

yes! ☺

Properties of Logarithms

• similar to properties of exponents → $a^m \cdot a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$

Notes

product: $\log_b m \cdot n = \log_b m + \log_b n$

quotient: $\log_b \frac{m}{n} = \log_b m - \log_b n$

Splitting the log into two...

Power: $\log_b m^n = n \cdot \log_b m$

$\log(m+n) \neq \log m \cdot \log n$

$\log(m-n) \neq \frac{\log m}{\log n}$

EXAMPLE 1 **Try It!** Prove a Property of Logarithms

1. Prove the ~~Quotient~~ Product Property of Logarithms.

Let $x = \log_b m$ ⇒ $b^x = m$

$y = \log_b n$ ⇒ $b^y = n$

$b^x \cdot b^y = m \cdot n$

$b^{x+y} = m \cdot n$ Conv to log

$\log_b m \cdot n = x + y$

$\log_b m \cdot n = \log_b m + \log_b n$

EXAMPLE 2 **Try It!** Expand Logarithmic Expressions

2. Use the properties of logarithms to expand each expression.

a. $\log_7\left(\frac{r^3t^4}{v}\right)$ ← fraction bar; Quotient

$\log_7(r^3t^4) - \log_7 v$ — Product

$\log_7 r^3 + \log_7 t^4 - \log_7 v$ — power

$3\log_7 r + 4\log_7 t - \log_7 v$

b. $\ln\left(\frac{7}{225}\right)$ ← Quotient

$\ln 7 - \ln 225$ $\langle^{15}_{15}$

$\ln 7 - \ln 15^2$ power

$\ln 7 - 2\ln 15$

EXAMPLE 3 **Try It!** Write Expressions as Single Logarithms — aka "Condense"

* unblock logs (coefficients) before unapplying properties...

* 2 Logs → 1 Log

3. Write each expression written as a single logarithm.

a. $5\log_2 c - 7\log_2 n$

un-power: $\log_2 c^5 - \log_2 n^7$ — un-quotient

$\log_2 \frac{c^5}{n^7}$

b. $2\ln 7 + \ln 2$

unpower: $\ln 7^2 + \ln 2$ — un-product

$\ln(7^2 \cdot 2)$ or $\ln 98$

HABITS OF MIND

Make Sense and Persevere Using the fact that $\log 2 \approx 0.3010$ and $\log 3 \approx 0.4771$, what is $\log 18$? Show how you know. MP.1

Clues.... $\log m \cdot n = \log m + \log n$

18 → 2 · 9 → 9 → 3 · 3

$\log 18 = \log(2 \cdot 3 \cdot 3)$

$= \log 2 + \log 3 + \log 3$

$= 0.3010 + .4771 + .4771 = 1.2552$

Notes

EXAMPLE 4 — Try It! Apply Properties of Logarithms

The pH of a solution is a measure of its concentration of hydrogen ions.
→ moles/liter [H+]

$pH = \log \frac{1}{[H^+]}$ base 10

OJ	Acid Rain	Clear Rain	Healthy Lake
pH=3	pH= 4.5	pH= 5.6	pH= 6.5

$10^{-4.5} \approx 0.0000316$ moles/L

4. What is the concentration of hydrogen ions in a liter of orange juice?

$pH = \log \frac{1}{[H^+]}$

$3 = \log \frac{1}{[H^+]}$ ← quotient

$3 = \log 1 - \log[H^+]$ $10^1 = 0$

$3 = -\log[H^+]$

$-3 = \log_{10}[H^+]$ log → exp

$10^{-3} = [H^+]$

$\frac{1}{1000} = [H^+]$

$\approx .0001$ moles/L

HABITS OF MIND

Generalize What types of numbers have logarithms that are negative? Explain. MP.8

EXAMPLE 5 — Try It! Evaluate Logarithmic Expressions by Changing the Base

ex) $\log_3 22$?

• change of base: base 3 → base 10

$\frac{\log 22}{\log 3} \approx 2.813588092...$

~~$\log_3 22 = x$~~
~~$3^x = 22$~~ Hmm...

Change of Base — base b → base a

$\log_b m = \frac{\log_a m}{\log_a b}$

5. What is the value of each logarithm to the nearest thousandth?

a. $\log_2 7$ — base 2 → 10

→ $\frac{\log 7}{\log 2} \approx 2.807$

b. $\log_5 3$

→ $\frac{\log 3}{\log 5} \approx 0.683$

EXAMPLE 6 — Try It! Use the Change of Base Formula

6. What is the solution to the equation $3^x = 15$? Express the solution as a logarithm and then evaluate. Round to the nearest thousandth.

$3^x = 15$ exp → log

$\log_3 15 = x$

$\frac{\log 15}{\log 3} = x \approx 2.465$

BAD: ~~$\frac{\log 15}{\log 3}$~~   ~~$\log \frac{15}{3}$~~

HABITS OF MIND

Use Appropriate Tools Why is the Change of Base Formula useful when evaluating a logarithm with a calculator? MP.5

base 10 & e

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How are the properties of logarithms used to simplify expressions and solve logarithmic equations?

2. **Vocabulary** While it is not necessary to change to base 10 when applying the Change of Base Formula, why is it common to do so?

3. **Error Analysis** Amanda claimed the expanded form of the expression $\log_4(c^2d^5)$ is $5\log_4 c + 5\log_4 d$. Explain the error Amanda made. MP.3

Do You KNOW HOW?

4. Use the properties of logarithms to expand the expression $\log_6\left(\frac{49}{5}\right)$.

quotient

Hmm...

$\log_6 49 - \log_6 5$

power

$\log_6 7^2 - \log_6 5$

$2\log_6 7 - \log_6 5$

5. Use the properties of logarithms to write the expression $5 \ln s + 6 \ln t$ as a single logarithm.

Condense

$5\ln s + 6\ln t$

unpower

$\ln s^5 + \ln t^6$

unproduct

$\ln s^5t^6$

6. Use the formula $\text{pH} = \log\frac{1}{[H^+]}$ to write an expression for the concentration of hydrogen ions, $[H^+]$, in a container of baking soda with a pH of 8.9. MP.4

$8.9 = \log\frac{1}{[H^+]}$

$8.9 = \log 1 - \log[H^+]$

$8.9 = -\log[H^+]$

$-8.9 = \log_{10}[H^+]$

$10^{-8.9} = [H^+]$

$[H^+] \approx .000000012589$

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