

6-6 Exponential and Logarithmic Equations

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MODEL & DISCUSS

A store introduces two new models of fitness trackers to its product line. A glance at the data is enough to see that sales of both types of fitness trackers are increasing. Unfortunately, the store has limited space for the merchandise. The manager decides that the store will sell both models until sales of TrackSmart exceed those of FitTracker.



A. Find an equation of an exponential that models the sales for each fitness tracker. Describe your method.

log form $\log_b x = y$ \leftrightarrow $b^y = x$ exp form

product $\log_b m \cdot n = \log_b m + \log_b n$

quotient $\log_b \frac{m}{n} = \log_b m - \log_b n$

Splitting
Logs
one \rightarrow two

B. Based on the equations that you wrote, determine when the store will stop selling FitTracker.

power $\log_b m^n = n \cdot \log_b m$

~~log neg #~~

HABITS OF MIND

Look for Relationships How do you know that the sales data is modeled by an exponential function? MP.7

Change of
base
 $b \rightarrow a$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

Property of Equality for Exponential $b > 0$ $b \neq 1$ $b^x = b^y$ iff $x = y$

Property of Equality for Exponential Equs

$b > 0$
 $b \neq 1$

$b^x = b^y \iff x = y$

manipulate bases to be the same...

ex 1) $\left(\frac{1}{2}\right)^{x+7} = 4^{3x}$
 want base 2
 $(2^{-1})^{x+7} = 2^{2(3x)}$
 $2^{-1(x+7)} = 2^{2(3x)}$
 same \rightarrow 2
 $-1(x+7) = 2(3x)$
 $-x-7 = 6x$
 $-7 = 7x$
 $-1 = x$

EXAMPLE 1 Try It! Solve Exponential Equations Using a Common Base

1. Solve each equation using a common base.

a. $25^{3x} = 125^{x+2}$
 base 5
 $(5^2)^{3x} = (5^3)^{x+2}$
 $2(3x) = 3(x+2)$
 $6x = 3x+6$
 $3x = 6$
 $x = 2$

b. $0.001 = 10^{6x}$
 base 10
 $\frac{1}{1000} = 10^{6x}$
 $\frac{1}{10^3} = 10^{6x}$
 $10^{-3} = 10^{6x}$
 $-3 = 6x$
 $-\frac{1}{2} = x$

EXAMPLE 2 Try It! Rewrite Exponential Equations Using Logarithms

2. Rewrite the equation $5^x = 12$ using logarithms.

~~$5^x = 12$~~
 ~~$\log_5 5^x = \log_5 12$~~
 $\log 5^x = \log 12$
 $x \cdot \frac{\log 5}{\log 5} = \frac{\log 12}{\log 5}$
 $x = \frac{\log 12}{\log 5}$
 $x \approx 1.544$

apply the \log_{10} to both sides

log form

also...
 $\log_5 12 = x$
 change of base

... Check:
 $5^{1.544} \stackrel{?}{=} 12$
 yes....

HABITS OF MIND

Communicate Precisely. In order to set the exponents of two exponential expressions equal to each other, what must be true about the exponential expression? MP.6

Property of Equality - If $x > 0$.

Property of Equality → If $x > 0$,
 then $\log x = \log y$ iff $x = y$

Notes

Log Both Sides
 ln

EXAMPLE 3

Try It! Solve Exponential Equations Using Logarithms

3. What is the solution to $2^{3x} = 7^{x+1}$?

$\log 2^{3x} = \log 7^{x+1}$
 $3x \log 2 = (x+1) \log 7$
 $3x \log 2 = x \log 7 + \log 7$
 $3x \log 2 - x \log 7 = \log 7$
 $x(3 \log 2 - \log 7) = \log 7$
 $x = \frac{\log 7}{3 \log 2 - \log 7}$
 $x \approx 14.573$

Check GC:
 $y_1 = 2^{3x}$
 $y_2 = 7^{x+1}$

EXAMPLE 4

Try It! Use an Exponential Model

4. About how many minutes does it take the fire to spread to cover 100 acres?



$160 = 4(1.8)^t$
 $6.276 = t$
 minutes

- undo operations
- log both sides

HABITS OF MIND

Use Structure Why is it useful to use logarithms to solve an exponential equation? © MP.7

- variable
- blah
- power property of LOGS

EXAMPLE 5

Try It! Solve Logarithmic Equations

5. Solve each equation.

a. $\log_5(x^2 - 45) = \log_5(4x)$

Prop of Equality of Logs

$x^2 - 45 = 4x$
 $x^2 - 4x - 45 = 0$
 $(x-9)(x+5) = 0$
 $x-9=0$ or $x+5=0$
 $x=9$ or $x=-5$

GC
 $\log_5 4(-5)$
 $\log_5 -20$

b. $\ln(-4x - 1) = \ln(4x^2)$

$-4x - 1 = 4x^2$
 $0 = 4x^2 + 4x + 1$
 $0 = (2x+1)(2x+1)$
 $x = -\frac{1}{2}$

PST

EXAMPLE 6

Try It! Solve Logarithmic and Exponential Equations by Graphing

6. Solve each equation by graphing. Round to the nearest thousandth.

a. $3(2)^{x+2} - 1 = 3 - x$

$y_1 = 3(2)^{x+2} - 1$
 $y_2 = 3 - x$
 $x \approx -1.205$

b. $\ln(3x - 1) = x - 5$

$y_1 = \ln(3x - 1)$
 $y_2 = x - 5$
 $x = 0.336$; 8.156

$3(2)^{x+2} - 1 = 3 - x$
 $3(2)^{x+2} = 4 - x$
 $(2)^{x+2} = \frac{4-x}{3}$
 $\log_2 \frac{4-x}{3} = x+2$
 UGH!

HABITS OF MIND

Generalize Summarize the procedure for solving a logarithmic equation. © MP.8

- Condense logs (if needed) → unblocked, no coefficients
- undo operations
- or apply Logs to both sides...

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do properties of exponents and logarithms help you solve equations?

2. **Vocabulary** Jordan claims that $x^2 + 3 = 12$ is an exponential equation. Is Jordan correct? Explain your thinking.

3. **Communicate Precisely** How can properties of logarithms help to solve an equation such as $\log_6(8x - 2)^3 = 12$? **MP.6**

Do You KNOW HOW?

Solve. Round to the nearest hundredth, if necessary. List any extraneous solutions.

4. $16^{3x} = 256^{x+1}$
 $16^{3x} = (16^2)^{x+1}$
 $3x = 2(x+1)$
 $3x = 2x + 2$
 $\frac{-2x}{x} = \frac{-2x}{-2x}$
 $x = 2$

• log both sides 5. $6^{x+2} = 4^x$
 $\log 6^{x+2} = \log 4^x$
 $(x+2)\log 6 = x\log 4$
 $x\log 6 + 2\log 6 = x\log 4$
 $x\log 6 - x\log 4 = -2\log 6$
 $x(\log 6 - \log 4) = -2\log 6$
 $x = \frac{-2\log 6}{\log 6 - \log 4}$
 $x \approx -8.838$

6. $\log_5(x^2 - 44) = \log_5(7x)$
Prop of Equality
 $x^2 - 44 = 7x$
 $x^2 - 7x - 44 = 0$
 $(x - 11)(x + 4) = 0$
 $x - 11 = 0$ $x + 4 = 0$
 $x = 11$ $x = -4$ **extraneous...**

7. $\log_2(3x - 2) = 4$
log → exp
 $2^4 = 3x - 2$
 $16 = 3x - 2$
 $18 = 3x$
 $6 = x$
exponentiate both sides...
 $2^{\log_2(3x-2)} = 2^4$
 $3x - 2 = 2^4$

8. $4^{2x} = 9^{x-1}$
log both sides
 $\log 4^{2x} = \log 9^{x-1}$
 $2x\log 4 = (x-1)\log 9$
 $2x\log 4 - x\log 9 = -\log 9$
 $x(2\log 4 - \log 9) = -\log 9$
 $x = \frac{-\log 9}{2\log 4 - \log 9} \approx -3.819$

9. A rabbit farm had 200 rabbits in 2015. The number of rabbits increases by 30% every year. How many rabbits are on the farm in 2031? **16 yrs**
 $y = a \cdot b^x$
 $y = 200(1.30)^x$
 $y = 200(1.30)^{16}$
 $= 13308$ rabbits
Hummm...
 They meant to solve for time given the final rabbit population?