PRACTICE & PROBLEM SOLVING

Scan for Multimedia

Practice Tutorial

Additional Exercises Available Online

UNDERSTAND

11. **Use Structure** Write a 2×2 matrix that does not have an inverse. Explain how you can tell that it does not have an inverse.

12. **Construct Arguments** Can a 2×3 matrix have an inverse? Explain.

13. **Error Analysis** Leah wants to find the area of a parallelogram with one vertex at the origin and defined by the vectors $\langle 3, 6\rangle$ and $\langle -4, -10\rangle$. Explain and correct Leah's error in finding the area of the parallelogram.

Let $T = \begin{bmatrix} 3 & -4 \\ 6 & -10 \end{bmatrix}$.

$A = \frac{1}{2}|\det T|$

$A = \frac{1}{2}|-6| = 3$

The area of the parallelogram is 3 square units. ✗

14. **Reason** Are $\begin{bmatrix} 8 & 4 \\ 4 & -2 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{16} & \frac{1}{8} \\ \frac{1}{8} & -\frac{1}{4} \end{bmatrix}$ inverses? Explain how you know.

15. **Higher Order Thinking** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find values of a, b, c, and d, where $A = A^{-1}$. (*Hint:* There are four distinct possible values.)

16. **Construct Arguments** Monisha said that to find det B, where $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, you can use the expression $ad - bc$ or the expression $bc - ad$. Is Monisha correct? Explain.

17. **Reason** Matrix A does not have an inverse. Find the value of b and explain how you know that this value for b is correct.

$A = \begin{bmatrix} -1 & b \\ 3 & 6 \end{bmatrix}$

PRACTICE

Find the inverse of each matrix. SEE EXAMPLE 1

18. $\begin{bmatrix} 10 & 2 \\ -5 & -3 \end{bmatrix}$

19. $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

Does each given matrix have an inverse? If so, find it. SEE EXAMPLE 2

20. $P = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$

21. $R = \begin{bmatrix} -2 & 8 & -5 \\ 3 & -11 & 7 \\ 9 & -34 & 21 \end{bmatrix}$

22. $Q = \begin{bmatrix} -6 & -9 \\ -4 & -6 \end{bmatrix}$

23. $S = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$

24. The matrix $\begin{bmatrix} 30 & 15 & 106 & 63 & 33 & 121 \\ 18 & 120 & 80 & 102 & 102 & 164 \\ 101 & 24 & 154 & 43 & 111 & 162 \end{bmatrix}$ was encoded using the matrix $A = \begin{bmatrix} -1 & 3 & 2 \\ 4 & 6 & -2 \\ 0 & 1 & 5 \end{bmatrix}$. What is the message? SEE EXAMPLE 3

25. The matrix $\begin{bmatrix} 49 & 145 & 173 & 124 & 76 & 215 \\ 18 & 50 & 62 & 46 & 30 & 78 \end{bmatrix}$ was encoded using the matrix $\begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$. What is the secret word?

Find the area of the triangle defined by the given vectors. SEE EXAMPLE 4

26. vectors $\langle 2, 10\rangle$ and $\langle -1, 5\rangle$

27. vectors $\langle 9, -3\rangle$ and $\langle -6, -1\rangle$

What is the area of a parallelogram with one vertex at the origin and defined by the given vectors? SEE EXAMPLE 5

28. vectors $\langle -4, 16\rangle$ and $\langle -2, 12\rangle$

29. vectors $\langle -5, 3\rangle$ and $\langle 7, -1\rangle$

PRACTICE & PROBLEM SOLVING

APPLY

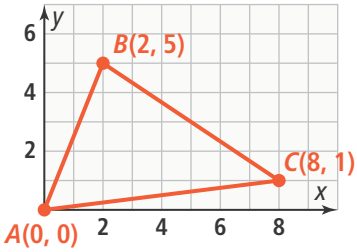
30. Reason A job title was hidden in the matrix

$$\begin{bmatrix} -34 & -29 & -35 & -23 & -19 & -92 \\ 123 & 93 & 114 & 219 & 66 & 153 \\ 97 & 26 & -137 & -83 & 11 & 16 \end{bmatrix}$$

using the encoding matrix $A = \begin{bmatrix} -4 & -2 & 1 \\ 0 & 3 & 6 \\ 5 & -8 & 2 \end{bmatrix}$.

Only those who discovered the title could apply. What job title was advertised?

31. Model With Mathematics The coordinate plane shows the location of a triangular park, where each unit on the grid represents 10 ft.

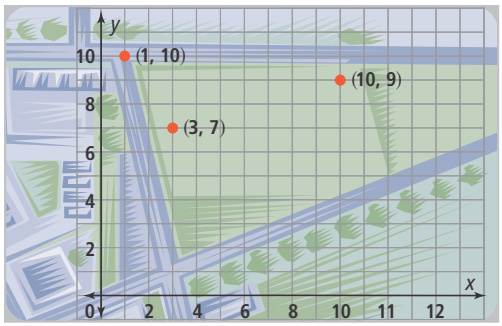


a. Use the vectors to write a matrix that represents the coordinates of the vertices of the triangular park.

b. What is the area of the triangular park?

c. A five-pound bag of grass seed covers about 300 square feet and costs $17.98. How much will it cost to cover the park with grass seed? Explain.

32. Make Sense and Persevere A city planner uses a coordinate plane to plan out a new neighborhood. Each grid square represents 4,000 square feet. A park, roughly in the shape of a parallelogram, is to be built so that one vertex of the parallelogram is located at (3, 7) on the planner's coordinate plane. Using this as an initial point, the points (10, 9) and (1, 10) are the terminal points of the two vectors that determine the parallelogram. What is the area of the park?



ASSESSMENT PRACTICE

33. Does the matrix have an inverse? Write *Yes* or *No*.

a. $\begin{bmatrix} -5 & 10 \\ -2 & -3 \end{bmatrix}$

b. $\begin{bmatrix} 15 & 2 \\ -12 & -3 \end{bmatrix}$

c. $\begin{bmatrix} 9 & -6 \\ 12 & 8 \end{bmatrix}$

d. $\begin{bmatrix} -3 & -6 \\ -6 & -12 \end{bmatrix}$

34. SAT/ACT The area of a triangle defined by the vectors $\langle 2, y \rangle$ and $\langle 4, 7 \rangle$ is 11 square units. What are the possible values of y?

Ⓐ 2 and 9 Ⓑ −2 and −9 Ⓒ −9 and 2 Ⓓ −2 and 9

35. Performance Task A credit card company encodes its issued credit card numbers when transmitting them electronically so that a customer's number is more secure. The company uses a 2×8 matrix to represent the 16 digits in the card number. The first column represents the first two digits, and so on.

Part A The matrix

$$\begin{bmatrix} 450 & 450 & 280 & 30 & 10 & 330 & 100 & 370 \\ 945 & 945 & 580 & 75 & 25 & 685 & 210 & 765 \end{bmatrix}$$

was encoded using the matrix $\begin{bmatrix} 10 & 40 \\ 25 & 80 \end{bmatrix}$.

What was the customer's credit card number?

Part B Create your own 16-digit credit card number and an encoding matrix, and encode your card number. Trade encoded card number matrices and the encoding matrix you used with a partner, and decode each other's numbers.

Part C The head of the company decides that a 2×2 encoding matrix is not secure enough and wants to institute a policy of using 3×3 encoding matrices. How will this affect the encoding process?