PRACTICE & PROBLEM SOLVING

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Practice Tutorial

Additional Exercises Available Online

UNDERSTAND

9. **Construct Arguments** Explain why the equation $X = BA^{-1}$ cannot be used to solve this matrix equation in the form $AX = B$.

$$\begin{bmatrix} -1 & 2 & -3 \\ 2 & -13 & 9 \\ -4 & 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2 \end{bmatrix}$$

10. **Communicate Precisely** Give an advantage of solving a system of linear equations using matrices instead of using the substitution or elimination method. (Assume you use technology to find the inverse matrix.)

11. **Error Analysis** Describe and correct the error a student made in solving the matrix equation $\begin{bmatrix} 5 & 9 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -26 \\ 16 \end{bmatrix}$.

$$\begin{bmatrix} 5 & 9 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -26 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} -26 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ -100 \end{bmatrix} \quad ✗$$

12. **Look for Relationships** Assume $a = c$ and $b = d$ for the matrix equation $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$. What is the relationship between e and f when there are infinitely many solutions? What is the relationship between e and f when there are no solutions?

13. **Higher Order Thinking** Write a system of linear equations in four variables, w, x, y, and z that has integer solutions. Then write a matrix equation to represent your system of equations. Finally, solve the matrix equation using technology to verify the integer solutions.

PRACTICE

Solve the matrix equation $A \cdot X = B$ for the given matrices. SEE EXAMPLE 1

14. $A = \begin{bmatrix} 8 & -7 \\ -6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -12 \end{bmatrix}$

15. $A = \begin{bmatrix} 2 & 8 & 4 \\ 1 & -1 & -3 \\ -3 & 2 & -9 \end{bmatrix}$ and $B = \begin{bmatrix} 26 \\ -2 \\ 37 \end{bmatrix}$

Express the system of linear equations as a matrix equation. SEE EXAMPLE 2

16. $8x + y = -1$
$-12x - 2y = 6$

17. $2x + 3y + 7z = 4$
$10x + 8y - 2z = -12$
$6x - y = 30$

Solve the following systems of linear equations using inverse matrices, if possible. SEE EXAMPLE 3

18. $-x + 2y = 8$
$-3x + 6y = -12$

19. $9x + 2y + 3z = 1$
$-8x - 3y - 4z = 1$
$12x + y - 2z = -17$

20. $-3x + 4y = -4$
$\frac{1}{2}x - 3y = -11$

21. $2x + \frac{2}{3}y + z = -8$
$x + 2y - \frac{1}{3}z = 6$
$-\frac{1}{2}x + 3y - 2z = 22$

22. Katrina makes bracelets and necklaces. Last week, she made 5 bracelets and 2 necklaces. This week, she made 3 bracelets and 5 necklaces. How many hours does it take Katrina to make one bracelet? How many hours does it take Katrina to make one necklace? SEE EXAMPLE 4

PRACTICE & PROBLEM SOLVING

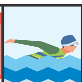
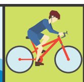
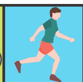
APPLY

23. Use Structure Luke had some quarters and dimes in his pocket. The quarters and dimes are worth \$2.55. He has 3 times as many quarters as dimes.

a. Write a matrix equation to find the number of quarters, x, and dimes, y, Luke has.

b. How many quarters and dimes does Luke have?

24. Make Sense and Persevere Malia is training for a triathlon. The table shows the number of hours she swam, biked, and ran and the total distance traveled on three different days. Write and solve a matrix equation to find Malia's average speed while swimming, biking, and running.

DAY				Total Distance (mi.)
1	$\frac{1}{3}$	1	2	32
2	$\frac{2}{3}$	$\frac{4}{5}$	1	22
3	1	2	$\frac{1}{2}$	37

25. Model With Mathematics Steve wants to mix three different types of cereal to create a mixture with 3,400 calories, 90 grams of protein, and 90 grams of fiber. The boxes of cereal show the number of calories, grams of protein, and grams of fiber in one serving of cereal A, B, and C. Write a matrix equation to represent this situation. How many servings of each type of cereal does Steve need to include in the mixture?

Calories: 300
Protein: 11g
Fiber: 8g

Calories: 300
Protein: 7g
Fiber:6g

Calories: 320
Protein: 8g
Fiber: 10g

ASSESSMENT PRACTICE

26. Which matrix equation has a unique solution? Select all that apply.

Ⓐ $\begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \end{bmatrix}$

Ⓑ $\begin{bmatrix} 2 & 9 \\ -3 & -8 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -46 \\ 36 \end{bmatrix}$

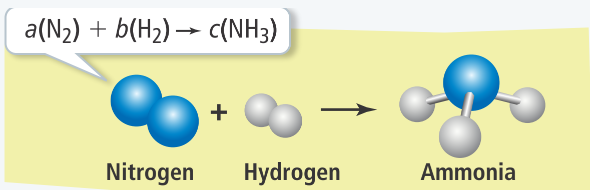
Ⓒ $\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$

Ⓓ $\begin{bmatrix} -4 & 2 \\ -14 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -7 \end{bmatrix}$

27. SAT/ACT The coordinates (x, y) of a point in a plane are the solution of the matrix equation $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$. In what quadrant is the point located?

Ⓐ I Ⓑ II Ⓒ III Ⓓ IV

28. Performance Task Nitrogen (N_2) and hydrogen (H_2) can react to form ammonia (NH_3). To write an equation for the reaction, the number of molecules of each element that are combined must equal the number of molecules of each element in the result. You can use matrices to figure out the coefficients that will balance the reaction. $a(N_2) + b(H_2) \rightarrow c(NH_3)$



Part A Let $c = 1$. Write a system of equations in a and b for this reaction. Make one equation to represent N and one equation to represent H, and think of the reaction arrow as an equal sign.

Part B Rewrite your system of equations as a matrix equation, and solve for a and b.

Part C Substitute the coefficients into the reaction equation. Multiply the equation by the least common denominator of all fractions so that all coefficients are whole numbers. Check that it is balanced.

Part D Use the same process to balance the reaction $a(Cr) + b(O_2) \rightarrow c(Cr_2O_3)$.