10-5 Reteach to Build Understanding

Inverse Matrices and Systems of Equations

1. Solve the matrix equation $A \cdot X = B$: $\begin{bmatrix} 12 & -3 \\ 16 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 144 \\ -64 \end{bmatrix}$.

a. Find the values for A, X, B, and A^{-1} (the inverse of A).

$A = \begin{bmatrix} 12 & \_\_ \\ \_\_ & 4 \end{bmatrix}$ $\quad A^{-1} = \begin{bmatrix} \frac{1}{24} & \frac{1}{32} \\ -\frac{1}{6} & \frac{1}{8} \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ \_\_ \end{bmatrix}$ $\quad B = \begin{bmatrix} 144 \\ \_\_ \end{bmatrix}$

b. Set up the equation $A^{-1} \cdot A \cdot X = A^{-1} \cdot B$.

$$\begin{bmatrix} \frac{1}{24} & \frac{1}{32} \\ -\frac{1}{6} & \frac{1}{8} \end{bmatrix} \cdot \begin{bmatrix} 12 & -3 \\ 16 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{24} & \frac{1}{32} \\ -\frac{1}{6} & \frac{1}{8} \end{bmatrix} \cdot \begin{bmatrix} 144 \\ -64 \end{bmatrix}$$

c. Any matrix A times its inverse equals the identity matrix I,

so $I \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{24} & \frac{1}{32} \\ -\frac{1}{6} & \frac{1}{8} \end{bmatrix} \cdot \begin{bmatrix} \_\_ \\ -64 \end{bmatrix}$.

d. Multiply. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{24} \cdot 144\right) + \left(\frac{1}{32} \cdot -64\right) \\ \left(-\frac{1}{6} \cdot 144\right) + \left(\frac{1}{8} \cdot -64\right) \end{bmatrix} = \begin{bmatrix} 6 + (-2) \\ -24 + (-8) \end{bmatrix} = \begin{bmatrix} 4 \\ \_\_ \end{bmatrix}$

e. The solution is $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -32 \end{bmatrix}$, so $x =$ _____ and $y =$ _____.

2. Solve the matrix equation $\begin{bmatrix} 3 & -9 \\ 1 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$.

a. Multiply each side by the inverse. $\begin{bmatrix} \frac{2}{3} & -1 \\ \frac{1}{9} & -\frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \_ & -9 \\ 1 & \_\_ \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \_ & -1 \\ \frac{1}{9} & \_ \end{bmatrix} \cdot \begin{bmatrix} 12 \\ \_ \end{bmatrix}$

b. Simplify. $\begin{bmatrix} \ \ \\ \ \ \end{bmatrix} = \begin{bmatrix} \left(\_ \cdot 12\right) + (-1 \cdot 0) \\ \left(\frac{1}{9} \cdot 12\right) + \left(\_ \cdot 0\right) \end{bmatrix} = \begin{bmatrix} \_ + 0 \\ \frac{12}{9} + \_ \end{bmatrix} = \begin{bmatrix} 8 \\ \frac{4}{3} \end{bmatrix}$

c. Solve. $x =$ _ and $y =$ _