TOPIC 1

Topic Review

TOPIC ESSENTIAL QUESTION

1. What are different ways in which functions can be used to represent and solve problems involving quantities?

Vocabulary Review

Choose the correct term to complete each sentence.

2. The _____ pairs every input in an interval with the same output value.
3. The point at which a function changes from increasing to decreasing is the _____ of the function.
4. A _____ of a function $y = af(x - h) + k$ is a change made to at least one of the values a, h, and k.
5. A _____ is the value of x when $y = 0$.
6. A _____ is defined by two or more functions, each over a different interval.

- step function
- piecewise-defined function
- minimum
- maximum
- system of linear equations
- transformation
- zero of the function

Concepts & Skills Review

LESSON 1-1 Key Features of Functions

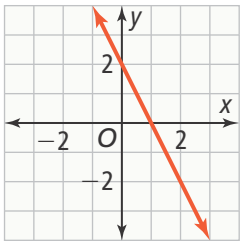
Quick Review

The domain of a function is the set of input values, or x-values. The range of a function is the set of output values, or y-values. These sets can be described using **interval notation** or **set-builder notation.**

A y-intercept is a point on the graph of a function where $x = 0$. An x-intercept is a point on the graph where $y = 0$. An x-intercept may also be a **zero of a function.**

Example

Find the zeros of the function. Then determine over what domain the function is positive or negative.

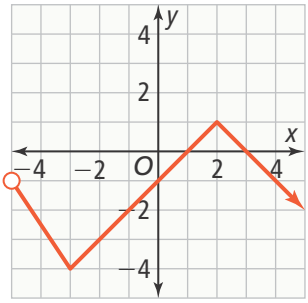


The point where the line crosses the x-axis is (1, 0), so $x = 1$ is a zero of the function. The function is positive on the interval $(-\infty, 1)$ and negative on the interval $(1, \infty)$.

Practice & Problem Solving

Identify the domain and range of the function in set-builder notation. Find the zeros of the function. Then determine for which values of x the function is positive and for which it is negative.

7.



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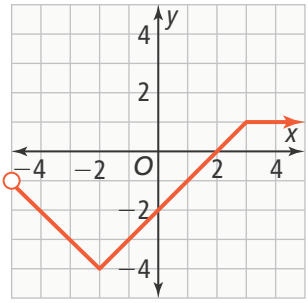


9. **Use Structure** Sketch a graph given the following key features.

   domain: (−5, 5); decreasing: (−3, 1); x-intercepts: −4, −2; positive: (−4, −2)

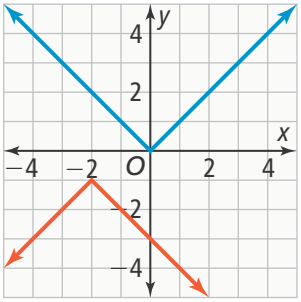
10. **Communicate Precisely** Jeffrey is emptying a 50 ft^3 container filled with water at a rate of 0.5 ft^3/min. Find and interpret the key features for this situation.

Quick Review

There are different types of **transformations** that change the graph of the parent function. A **translation** shifts each point on a graph the same distance and direction. A **reflection** maps each point to a new point across a given line. A **stretch** or a **compression** increases or decreases the distance between the points of a graph and a given line by the same factor.

Example

Graph the parent function $f(x) = |x|$ and $g(x) = -|x + 2| - 1$. Describe the transformation.



Multiplying the absolute value expression by −1 indicates a reflection over the *x*-axis.

Adding 2 to *x* indicates a translation 2 units to the left and subtracting 1 from the absolute value expression indicates a translation 1 unit down.

So the graph of *g* is a reflection of the graph of the parent function *f* over the *x*-axis, and then a translation 2 units left and 1 unit down.

Practice & Problem Solving

Graph each function as a translation of its parent function, *f*.

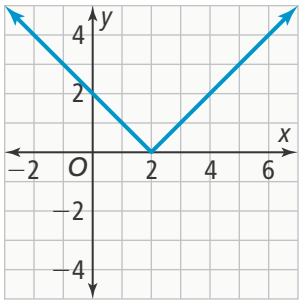
11. $g(x) = |x| - 7$

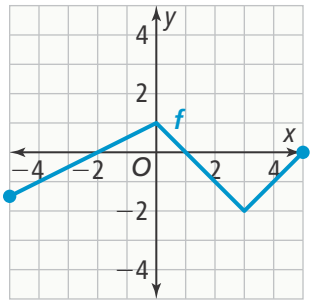
12. $g(x) = x^2 + 5$

Graph the function, *g*, as a reflection of the graph of *f* across the given axis.

13. across the *x*-axis

14. across the *y*-axis





15. **Look for Relationships** Describe the effect of a vertical stretch by a factor greater than 1 on the graph of the absolute value function. How is that different from the effect of a horizontal stretch by the same factor?

16. **Use Structure** Graph the function that is a vertical stretch by a factor of 3.5 of the parent function $f(x) = |x|$.

17. **Use Structure** Graph the function that is a horizontal translation 1 unit to the right of the parent function $f(x) = x^2$.

Quick Review

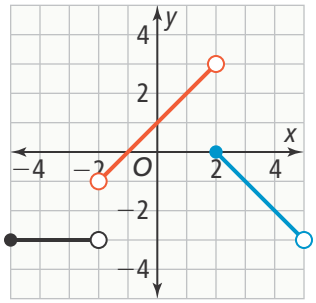
A **piecewise-defined function** is a function defined by two or more function rules over different intervals. A **step function** pairs every number in an interval with a single value. The graph of a step function can look like the steps of a staircase.

Example

Graph the function.

$$y = \begin{cases} -3, \text{ if } -5 \le x < -2 \\ x + 1, \text{ if } -2 < x < 2 \\ -x + 2, \text{ if } 2 \le x < 5 \end{cases}$$

State the domain and range. Identify whether the function is increasing, constant, or decreasing on each interval of the domain.



Graph the function.

Domain: $-5 \le x < -2$ and $-2 < x < 5$

Range: $-3 \le y < 3$

Increasing when $-2 < x < 3$

Constant when $-5 \le x < -2$

Decreasing when $2 \le x < 5$

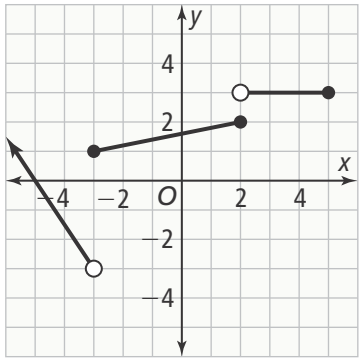
Practice & Problem Solving

Graph each function.

18. $y = \begin{cases} -3, \text{ if } -4 \le x < -2 \\ -1, \text{ if } -2 \le x < 0 \\ 1, \text{ if } 0 \le x < 2 \\ 3, \text{ if } 2 \le x < 4 \end{cases}$

19. $y = \begin{cases} 2x + 5, \text{ if } x < -3 \\ -x - 2, \text{ if } -3 \le x < 1 \\ x - 3, \text{ if } x \ge 1 \end{cases}$

20. What rule defines the function in the following graph?



21. **Generalize** Can every transformation of the absolute value function also be written as a piecewise-defined function? Explain.

22. **Model With Mathematics** A coach is trying to decide how many new uniforms to purchase for a softball team. If the coach orders more than 10 uniforms, the cost for the extra uniforms is 0.75 times the normal cost per uniform of $120. Write a piecewise-defined function that gives the cost C, in dollars, in terms of the number of uniforms n the coach purchases. Determine how much the coach will pay for 18 uniforms.

LESSON 1-4 Arithmetic Sequences and Series

Quick Review

An **arithmetic sequence** is a sequence with a constant difference between consecutive terms. This difference is known as the **common difference.**

recursive definition: $a_n = \begin{cases} a_1, \text{ if } n = 1 \\ a_{n-1} + d, \text{ if } n > 1 \end{cases}$

explicit definition: $a_n = a_1 + (n - 1)d$

A **finite arithmetic series** is the sum of all the numbers in an arithmetic sequence.

Example

Given the sequence 22, 17, 12, 7, …, write the explicit formula. Then find the 6th term.

$d = -5$ ····· Find the common difference.

$a_n = 22 + (n - 1)(-5)$ ····· Substitute 22 for a_1 and -5 for d.

$a_n = 22 - 5(n - 1)$ ····· Simplify.

$a_6 = 22 - 5(6 - 1)$ ····· Substitute 6 for n.

$a_6 = -3$ ····· Solve for the 6th term.

Practice & Problem Solving

What is the common difference and the next term in the arithmetic sequence?

23. 3, 15, 27, 39, …

24. 19, 13, 7, 1, …

What are the recursive and explicit functions for each sequence?

25. 5, 9, 13, 17, 21, …

26. 25, 18, 11, 4, −3, …

Find the sum of an arithmetic sequence with the given number of terms and values of a_1 and a_n.

27. 8 terms, $a_1 = 2$, $a_8 = 74$

28. 12 terms, $a_1 = 87$, $a_{12} = 10$

What is the value of each of the following series?

29. $\sum_{n=1}^{9} (1 + 3n)$

30. $\sum_{n=1}^{6} (5n - 2)$

31. **Make Sense and Persevere** Cubes are stacked in the shape of a pyramid. The top row has 1 cube, the second row has 3, and the third row has 5. If there are 9 rows of cubes, how many cubes were used to make the front of the pyramid?

LESSON 1-5 Solving Equations and Inequalities by Graphing

Quick Review

To solve an equation by graphing, write two new equations by setting y equal to each expression in the original equation. Approximate coordinates of any points of intersection. The x-values of these points are the solutions to the equation. You can also solve equations using tables or graphing technology.

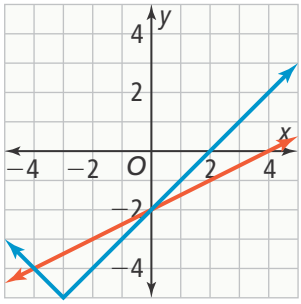
Example

Solve $|x + 3| - 5 = \frac{1}{2}x - 2$ by graphing.

Graph $y = |x + 3| - 5$ and $y = \frac{1}{2}x - 2$.

It appears that $x = -4$ and $x = 0$ are solutions.

Confirm the solutions by substituting into the original equation.



Practice & Problem Solving

Use a graph to solve each equation.

32. $-x + 2 = x^2$

33. $\frac{1}{4}|x + 3| = 2$

Use a graph to solve each inequality.

34. $x^2 + 2x - 3 > 0$

35. $x^2 - 7x - 8 < 0$

36. **Construct Arguments** Is graphing always the most convenient method for solving an equation? Why or why not?

37. **Model With Mathematics** A truck is traveling 30 mi ahead of a car at an average rate of 55 mph. The car is traveling at a rate of 63 mph. Let x represent the number of hours that the car and truck travel. Write an inequality to determine at what times the car will be ahead of the truck and graph the inequality to solve.

LESSON 1-6 Linear Systems

Quick Review

A **system of linear equations** is a set of two or more equations using the same variables. The **solution of a system of linear equations** is the set of all ordered coordinates that simultaneously make all equations in the system true. A **system of linear inequalities** is a set of two or more inequalities using the same variables.

Example

Solve the system. $\begin{cases} -4x + 4y = 16 \\ -x + 2y = 10 \end{cases}$

$x = 2y - 10$ ····· Solve the second equation for x.

$-4(2y - 10) + 4y = 16$ ····· Substitute $2y - 10$ for x. Solve for y.
$y = 6$

$x = 2(6) - 10$ ····· Substitute 6 for y in the equation $x = 2y - 10$.
$x = 2$

Practice & Problem Solving

Solve each system of equations.

38. $\begin{cases} y = 2x + 5 \\ 2x + 4y = 10 \end{cases}$ **39.** $\begin{cases} y = 2x - 6 \\ 6x + y = 10 \end{cases}$

40. **Use Structure** Write a linear system in two variables that has infinitely many solutions.

41. **Model With Mathematics** It takes Leo 12 h to make a table and 20 h to make a chair. In 8 wk, Leo wants to make at least 5 tables and 8 chairs to display in his new shop. Leo works 40 h a week. Write a system of linear inequalities relating the number of tables x and the number of chairs y Leo will be able to make. List two different combinations of tables and chairs Leo could have to display at the opening of his new shop.

LESSON 1-7 Solving Linear Systems Using Matrices

Quick Review

You can solve systems with matrices. A matrix is a rectangular array of numbers, usually shown inside square brackets. Row operations can be applied to a matrix to create an equivalent matrix and can be used to write the matrix in **reduced row echelon form.**

Example

Solve the system $\begin{cases} -2x + 8y = 10 \\ 4x - 3y = 6 \end{cases}$ **using a matrix.**

$\left[\begin{array}{cc|c} -2 & 8 & 10 \\ 4 & -3 & 6 \end{array}\right]$ ····· Write the system in matrix form.

$\left[\begin{array}{cc|c} 1 & -4 & -5 \\ 4 & -3 & 6 \end{array}\right]$ ····· Divide row_1 by -2.

$\left[\begin{array}{cc|c} 1 & -4 & -5 \\ 0 & 13 & 26 \end{array}\right]$ ····· Multiply row_1 by -4, and add to row_2.

$\left[\begin{array}{cc|c} 1 & -4 & -5 \\ 0 & 1 & 2 \end{array}\right]$ ····· Divide row_2 by 13.

$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$ ····· Multiply row_2 by 4, and add to row_1.

The solution to the system of linear equations is $x = 3$ and $y = 2$.

Practice & Problem Solving

42. Write the matrix that represents the system of equations and find the reduced row echelon form. $\begin{cases} 4x + 8y = 12 \\ -2x - 6y = 32 \end{cases}$

43. Write a system of equations represented by the matrix. $\left[\begin{array}{cc|c} 5 & 2 & 6 \\ 6 & -7 & -4 \end{array}\right]$

44. **Communicate Precisely** Why is it important to write equations in standard form before entering the coefficients into a matrix?

45. **Model With Mathematics** A trivia game consists of three types of questions in three different colors: red, white, and blue. Each type of question is worth a different number of points. Holly answered 4 red, 1 white, and 1 blue question correctly and earned 23 points. Jung answered 5 white and 1 blue question correctly and earned 35 points. Rochelle answered 2 red and 3 white questions, and earned 19 points. How many points is each color worth?